Matrices

Equal Matrices

Solve for x and y.

1. $\begin{bmatrix} 2x \\ 2x+3y \end{bmatrix} = \begin{bmatrix} y \\ 12 \end{bmatrix}$ $x = \frac{3}{2}$ $y = 3$

2. $\begin{bmatrix} 3x+y \\ x-2y \end{bmatrix} = \begin{bmatrix} x+3 \\ y-2 \end{bmatrix}$ $x = 1$ $y = 1$

3. $\begin{bmatrix} 2x & 3 & 3z \end{bmatrix} = \begin{bmatrix} 5 & 3y & 9 \end{bmatrix}$ $x = \frac{5}{2}$ $y = 1$ $z = 3$

Adding and Subtracting Matrices

Only matrices with the same dimensions can be added or subtracted.
The resulting matrix has same dimensions.

Examples

1. $\begin{bmatrix} -2 & 0 & 4 \\ 3 & -10 & 12 \\ 3 & -2 & -2 \end{bmatrix} + \begin{bmatrix} -4 & 6 & 0 \\ -15 & 2 & -4 \\ 6 & 7 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 6 & 4 \\ -12 & -8 & 8 \\ 9 & 5 & -1 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 8 \\ 0 & 2 & 4 \end{bmatrix}$ cannot be added

3. $\begin{bmatrix} 2 & -18 \\ 20 & -5 \end{bmatrix} - \begin{bmatrix} -4 & 2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -20 \\ 25 & -6 \end{bmatrix}$

4. $\begin{bmatrix} -3 & 10 & 2 \\ -10 & 8 & -6 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \\ -8 & -10 & -4 \end{bmatrix} = \begin{bmatrix} -8 & 6 & -1 \\ -12 & 7 & -6 \\ 8 & 11 & 4 \end{bmatrix}$

Matrices

Scalar Multiplication

Examples:

1. $-2\begin{bmatrix} 7 & -1 & 0 \end{bmatrix}$

$\begin{bmatrix} -14 & 2 & 0 \end{bmatrix}$

2. $4\begin{bmatrix} -2 & 0 \\ 4 & -5 \end{bmatrix}$

$\begin{bmatrix} -8 & 0 \\ 16 & -20 \end{bmatrix}$

3. $-5\begin{bmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \\ -7 & 8 & -9 \end{bmatrix}$ $\begin{bmatrix} -5 & -10 & 15 \\ 20 & 25 & -30 \\ 35 & -40 & 45 \end{bmatrix}$

Matrix Multiplication

Multiply rows times columns
**You can only multiply if the number of columns in the 1st matrix is equal to the number of rows in the 2nd matrix.

1. $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix} = \begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ cannot multiply

3. $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 2 \\ 2 & 6 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 8 \end{bmatrix}$ $\begin{bmatrix} 1x + 2y - 1z = 1 \\ 1x + 3y + 2z = 7 \\ 2x + 6y + 1z = 8 \end{bmatrix}$

4. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 4 & 3 \end{bmatrix}$

Matrices

Determinants

Determinant of a 2x2 matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - cb$$

Find the determinant of each:

1. $\begin{vmatrix} -5 & -7 \\ 11 & 8 \end{vmatrix}$ 37   2. $\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix}$ 17   3. $\begin{vmatrix} 10 & -2 \\ 0 & -3 \end{vmatrix}$ -30

**To find a determinant you must have a square matrix!!

Determinant for a 3x3 matrix: Expansion by minors
*minor of an element is the determinant formed when the row and the column containing that element are deleted!

Examples:

1. $\begin{vmatrix} -2 & 3 & 8 \\ 6 & 7 & -1 \\ -4 & 5 & 9 \end{vmatrix}$

$$-2\begin{vmatrix} 7 & -1 \\ 5 & 9 \end{vmatrix} - 3\begin{vmatrix} 6 & -1 \\ -4 & 9 \end{vmatrix} + 8\begin{vmatrix} 6 & 7 \\ -4 & 5 \end{vmatrix}$$

$$-2(68) - 3(50) + 8(58)$$

$$-136 - 150 + 464$$

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2. $\begin{vmatrix} 5 & -1 & 2 \\ 2 & -3 & 5 \\ 3 & 2 & -3 \end{vmatrix}$

$$+1\begin{vmatrix} 2 & 5 \\ 3 & -3 \end{vmatrix} - 3\begin{vmatrix} 5 & 2 \\ 3 & -3 \end{vmatrix} - 2\begin{vmatrix} 5 & 2 \\ 2 & 5 \end{vmatrix}$$

$$+1(-21) - 3(-21) - 2(21)$$

$$-21 + 63 - 42$$

0

3. $\begin{vmatrix} -1 & 0 & 4 \\ 2 & -2 & 2 \\ 3 & 0 & -1 \end{vmatrix}$

$$0\begin{vmatrix} 2 & 2 \\ 3 & -1 \end{vmatrix} - 2\begin{vmatrix} -1 & 4 \\ 3 & -1 \end{vmatrix} \; 0\begin{vmatrix} -1 & 4 \\ 2 & 2 \end{vmatrix}$$

$$-2(-11)$$

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Determinant for a 3x3 matrix: Diagonal Method
Examples:

1. $\begin{vmatrix} -2 & 3 & 8 \\ 6 & 7 & -1 \\ -4 & 5 & 9 \end{vmatrix}$ $(-2\cdot 7\cdot 9)+(3\cdot -1\cdot -4)+(8\cdot 6\cdot 5)-(-4\cdot 7\cdot 8)-(5\cdot -1\cdot -2)$
$-(9\cdot 6\cdot 3)$
$-126+12+240+224-10-162$

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2. $\begin{vmatrix} 5 & -1 & 2 \\ 2 & -3 & 5 \\ 3 & 2 & -3 \end{vmatrix}$ $(5\cdot -3\cdot -3)+(-1\cdot 5\cdot 3)+(2\cdot 2\cdot 2)-(3\cdot -3\cdot 2)-(2\cdot 5\cdot 5)$
$-(\cdot 3\cdot 2\cdot -1)$
$45-15+8+18-50-6$

0

3. $\begin{vmatrix} -1 & 0 & 4 \\ 2 & -2 & 2 \\ 3 & 0 & -1 \end{vmatrix}$ $(-1\cdot -2\cdot -1)+(0\cdot 2\cdot 3)+(4\cdot 2\cdot 0)-(3\cdot -2\cdot 4)-(0\cdot 2\cdot -1)$
$-(-1\cdot 2\cdot 0)$
$-2+0+0+24-0-0$

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Solving Systems using Cramer's Rule

Cramer's Rule:

using matrices to solve a system of equations
find the determinant of the matrix
then substitute the answers to equations for the first variable and find determinant of that
repeat third step until determinant of each variable has been found

answers expressed in $\frac{\text{determinant of variable}}{\text{determinant}}$ form

Identity and Inverse Matrices

Identity Matrix

this occurs when matrix A is multiplied by its inverse

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse of a 2x2 matrix

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Examples:

1. $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} = -1 \begin{bmatrix} 1 & -3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$

   det A = (5x1) - (2x3)

   -1

2. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$

   det A = (1x4) - (3x2)

   -2

3. $\begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 7 & 5 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 5/14 \\ 0 & 1/7 \end{bmatrix}$

   det A = (2x7) - (0x5)

   14

???Is there ever a square matrix that does not have an inverse???

yes called noninvertible
this happens when the determinant is zero

Solving Systems of equations using matrices

- Coefficient Matrix
  matrix formed by coefficients in a system of equations
- Variable Matrix
  matrix formed by variables in a system of equations
- Constant Matrix
  matrix formed by answers in a system of equations

Example: $\begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

Steps to solving:

1. find the inverse of the coefficient matrix
2. multiply both sides of equation by the inverse.
3. check answers by substituting x and y into original equations.

Example: Solve using the inverse matrix method

$4x - 12y = 7$

$x + 6y = 9$

$$\begin{bmatrix} 4 & -12 \\ 1 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$$

$$\frac{1}{4\cdot 6 - 1\cdot -12} \begin{bmatrix} 6 & 12 \\ -1 & 4 \end{bmatrix} = \frac{1}{24+12} = \frac{1}{36}\begin{bmatrix} 6 & 12 \\ -1 & 4 \end{bmatrix}$$

$$\frac{1}{36}\begin{bmatrix} 6 & 12 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 4 & -12 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{36}\begin{bmatrix} 6 & 12 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 7 \\ 9 \end{bmatrix}$$

$$= \frac{1}{36}\begin{bmatrix} 150 \\ 29 \end{bmatrix}$$

$$\left(\frac{150}{36}, \frac{29}{36}\right)$$

$$(x, y)$$