1.1 The Pythagorean Theorem

1. $6^2+9^2?13^2 \rightarrow 36+81?169 \rightarrow 117<169$ The triangle is obtuse.
2. $9^2+10^2?11^2 \rightarrow 81+100?121 \rightarrow 181>121$ The triangle is acute.
3. $16^2+30^2?34^2 \rightarrow 256+900?1156 \rightarrow 1156=1156$ This is a right triangle.
4. $20^2+23^2?40^2 \rightarrow 400+529?1600 \rightarrow 929<1600$ The triangle is obtuse.
5. These lengths cannot make up the sides of a triangle. $11+16<29$
6. $\left(2\sqrt{6}\right)^2+\left(6\sqrt{3}\right)^2?\left(2\sqrt{33}\right)^2 \rightarrow (4\cdot 6)+(36\cdot 3)?(4\cdot 33) \rightarrow 24+108?132 \rightarrow 132=132$ This is a right triangle.
7.

$$\begin{aligned} 7^2+x^2&=18^2\\ 49+x^2&=324\\ x^2&=275\\ x&=\sqrt{275}=5\sqrt{11} \end{aligned}$$

8.

$$\begin{aligned} 5^2+\left(5\sqrt{3}\right)^2&=x^2\\ 25+(25\cdot 3)&=x^2\\ 25+75&=x^2\\ 100&=x^2\\ 10&=x \end{aligned}$$

9. Both legs are 11.

$$\begin{aligned} 11^2+11^2&=x^2\\ 121+121&=x^2\\ 242&=x^2\\ \sqrt{242}&=x\\ 11\sqrt{2}&=x \end{aligned}$$

10. Plug $n^2-m^2, 2nm, n^2+m^2$ into the Pythagorean Theorem.

$$\begin{aligned} (n^2-m^2)^2+(2nm)^2&=(n^2+m^2)^2\\ n^4-2n^2m^2+m^4+4n^2m^2&=n^4+2n^2m^2+m^4\\ -2n^2m^2+4n^2m^2&=2n^2m^2\\ 4n^2m^2&=4n^2m^2 \end{aligned}$$

11. (a) (5, -6) and (18, 3)

$$\begin{aligned} d &= \sqrt{(5-18)^2+(-6-3)^2} \\ &= \sqrt{(-13)^2+(-9)^2} \\ &= \sqrt{169+81} \\ &= \sqrt{250} \\ &= 5\sqrt{10} \end{aligned}$$

(b) $\left(\sqrt{3},\ -\sqrt{2}\right)$ and $\left(-2\sqrt{3},\ 5\sqrt{2}\right)$

$$\begin{aligned} d &= \sqrt{\left(\sqrt{3}-\left(-2\sqrt{3}\right)\right)^2+\left(-\sqrt{2}-5\sqrt{2}\right)^2} \\ &= \sqrt{\left(3\sqrt{3}\right)^2+\left(-6\sqrt{2}\right)^2} \\ &= \sqrt{(9\cdot 3)+(36\cdot 2)} \\ &= \sqrt{27+72} \\ &= \sqrt{99} = 3\sqrt{11} \end{aligned}$$