1.3 Basic Trigonometric Functions

1. Answers:
   - $\sin A = \frac{9}{15} = \frac{3}{5}$
   - $\cos A = \frac{12}{15} = \frac{4}{5}$
   - $\tan A = \frac{9}{12} = \frac{3}{4}$
   - $\csc A = \frac{15}{9} = \frac{5}{3}$
   - $\sec A = \frac{15}{12} = \frac{5}{4}$
   - $\cot A = \frac{12}{9} = \frac{4}{3}$
2. The hypotenuse is 17 $\left(\sqrt{15^2+8^2} = \sqrt{225+64} = \sqrt{289} = 17\right)$.
   - $\sin T = \frac{15}{17}$
   - $\cos T = \frac{8}{17}$
   - $\tan T = \frac{15}{8}$
   - $\csc T = \frac{17}{15}$
   - $\sec T = \frac{17}{8}$
   - $\cot T = \frac{8}{15}$
3. Answers:

   a. The hypotenuse is 13 $\left(\sqrt{5^2+12^2} = \sqrt{25+144} = \sqrt{169} = 13\right)$.

   b. $\sin X = \frac{12}{13}, \cos X = \frac{5}{13}, \tan X = \frac{12}{5}, \csc X = \frac{13}{12}, \sec X = \frac{13}{5}, \cot X = \frac{5}{12}$

   c. $\sin Z = \frac{5}{13}, \cos Z = \frac{12}{13}, \tan Z = \frac{5}{12}, \csc Z = \frac{13}{5}, \sec Z = \frac{13}{12}, \cot Z = \frac{12}{5}$
4. From #3, we can conclude that:
   - $\sin X = \cos Z$
   - $\cos X = \sin Z$
   - $\tan X = \cot Z$
   - $\cot X = \tan Z$
   - $\csc X = \sec Z$
   - $\sec X = \csc Z$
   - Yes, this can be generalized for all complements.
5. The hypotenuse is $2\sqrt{2}$. Each angle is 45°, so the sine, cosine, and tangent are the same for both angles.
   - $\sin 45^\circ = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   - $\cos 45^\circ = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   - $\tan 45^\circ = \frac{2}{2} = 1$
6. If the legs are length x, then the hypotenuse is $x\sqrt{2}$. For 45°, the sine, cosine, and tangent are:
   - $\sin 45^\circ = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   - $\cos 45^\circ = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   - $\tan 45^\circ = \frac{x}{x} = 1$

   This tells us that regardless of the length of the sides of an isosceles right triangle, the sine, cosine and tangent of 45° are always the same.

7. If the hypotenuse is 10, then the short leg is 5 and the long leg is $5\sqrt{3}$. Recall, that 30° is opposite the short side, or 5, and 60° is opposite the long side, or $5\sqrt{3}$.
   - $\sin 30^\circ = \frac{5}{10} = \frac{1}{2}$
   - $\cos 30^\circ = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$
   - $\tan 30^\circ = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
   - $\sin 60^\circ = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$
   - $\cos 60^\circ = \frac{5}{10} = \frac{1}{2}$
   - $\tan 60^\circ = \frac{5\sqrt{3}}{5} = \sqrt{3}$
8. If the short leg is x, then the long leg is $x\sqrt{3}$ and the hypotenuse is $2x$. 30° is opposite the short side, or x, and 60° is opposite the long side, or $x\sqrt{3}$.
   - $\sin 30^\circ = \frac{x}{2x} = \frac{1}{2}$
   - $\cos 30^\circ = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$
   - $\tan 30^\circ = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
   - $\sin 60^\circ = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$
   - $\cos 60^\circ = \frac{x}{2x} = \frac{1}{2}$
   - $\tan 60^\circ = \frac{x\sqrt{3}}{x} = \sqrt{3}$

   This tells us that regardless of the length of the sides of a $30-60-90$ triangle, the sine, cosine and tangent of 30° and 60° are always the same. Also, $\sin 30^\circ = \cos 60^\circ$ and $\cos 30^\circ = \sin 60^\circ$.}}
9. If $\sin A = \frac{9}{41}$, then the opposite side is $9x$ (some multiple of 9) and the hypotenuse is $41x$. Therefore, working with the Pythagorean Theorem would give us the length of the other leg. Also, we could notice that this is a Pythagorean Triple and the other leg is $40x$.