1.6 Applying Trig Functions to Angles of Rotation

1. The radius of the circle is 5.

$$\cos\theta = \frac{3}{5} \qquad \sec\theta = \frac{5}{3}$$
$$\sin\theta = \frac{-4}{5} \qquad \csc\theta = \frac{5}{-4}$$
$$\tan\theta = \frac{-4}{3} \qquad \cot\theta = \frac{3}{-4}$$

2. The radius of the circle is 13.

$$\cos\theta = \frac{-5}{13} \qquad \sec\theta = \frac{13}{-5}$$
$$\sin\theta = \frac{-12}{13} \qquad \csc\theta = \frac{13}{-12}$$
$$\tan\theta = \frac{-12}{-5} = \frac{12}{5} \qquad \cot\theta = \frac{-5}{-12} = \frac{5}{12}$$

3. If $\tan\theta = -\frac{2}{3}$, it must be in either Quadrant II or IV. Because $\cos\theta > 0$, we can eliminate Quadrant II. So, this means that the 3 is negative. (All Students Take Calculus) From the Pythagorean Theorem, we find the hypotenuse:

$$\begin{aligned} 2^2 + (-3^2) &= c^2 \\ 4 + 9 &= c^2 \\ 13 &= c^2 \\ \sqrt{13} &= c \end{aligned}$$

Because we are in Quadrant IV, the sine is negative. So, $\sin\theta = -\frac{2}{\sqrt{13}}$ or $-\frac{2\sqrt{13}}{13}$ (Rationalize the denominator)

4. If $\csc\theta = -4$, then $\sin\theta = -\frac{1}{4}$, sine is negative, so θ is in either Quadrant III or IV. Because $\tan\theta > 0$, we can eliminate Quadrant IV, therefore θ is in Quadrant III. From the Pythagorean Theorem, we can find the other leg:

$$\begin{aligned} a^2 + (-1)^2 &= 4^2 \\ a^2 + 1 &= 16 \\ a^2 &= 15 \\ a &= \sqrt{15} \end{aligned}$$

$$\text{So, } \cos\theta = -\frac{\sqrt{15}}{4}, \sec\theta = -\frac{4}{\sqrt{15}} \text{ or } -\frac{4\sqrt{15}}{15}$$
$$\tan\theta = -\frac{1}{\sqrt{15}} \text{ or } \frac{\sqrt{15}}{15}, \cot\theta = \sqrt{15}$$

5. If the terminal side of θ is on (2, 6) it means θ is in Quadrant I, so sine, cosine and tangent are all positive.

From the Pythagorean Theorem, the hypotenuse is:

$$\begin{aligned}2^2+6^2&=c^2\\4+36&=c^2\\40&=c^2\\\sqrt{40}=2\sqrt{10}&=c\end{aligned}$$

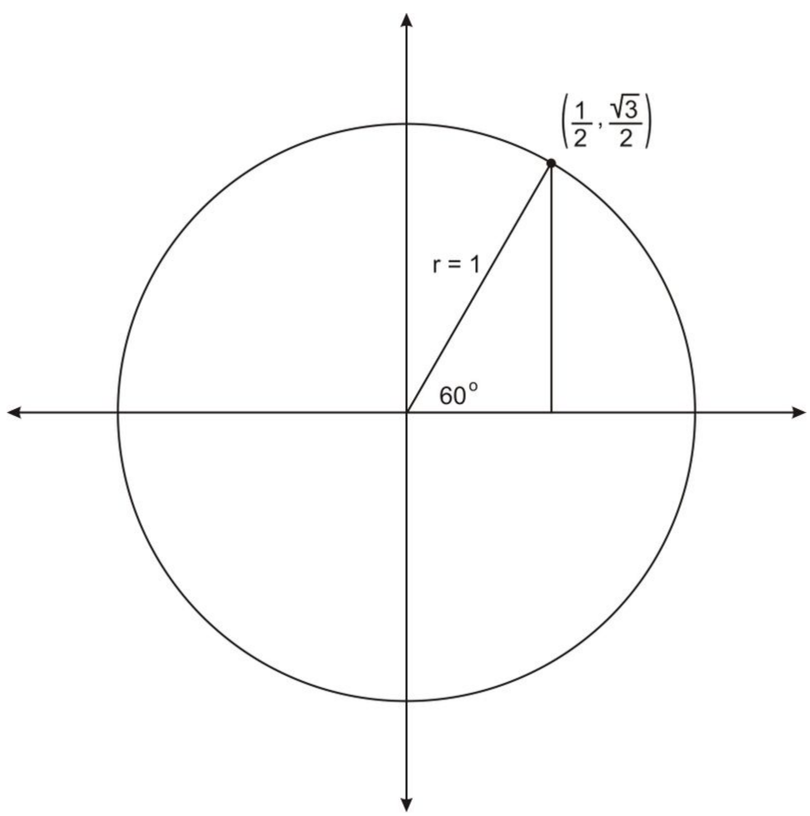
Therefore, $\sin\theta=\frac{6}{2\sqrt{10}}=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}, \cos\theta=\frac{2}{2\sqrt{10}}=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10}$ and $\tan\theta=\frac{6}{2}=3$

6.

$$\cos 270=0 \qquad \sec 270=undefined$$
$$\sin 270=-1 \qquad \csc 270=\frac{1}{-1}=-1$$
$$\tan 270=undefined \qquad \cot 270=0$$

7. Answers:

a. The triangle is equiangular because all three angles measure 60 degrees. Angle DAB measures 60 degrees because it is the sum of two 30 degree angles.
b. BD has length 1 because it is one side of an equiangular, and hence equilateral, triangle.
c. BC and CD each have length $\frac{1}{2}$, as they are each half of BD. This is the case because Triangle ABC and ADC are congruent.
d. We can use the Pythagorean theorem to show that the length of AC is $\frac{\sqrt{3}}{2}$. If we place angle BAC as an angle in standard position, then AC and BC correspond to the x and y coordinates where the terminal side of the angle intersects the unit circle. Therefore the ordered pair is $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$.
e. If we draw the angle 60° in standard position, we will also obtain a $30-60-90$ triangle, but the side lengths will be interchanged. So the ordered pair for 60° is $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$.



8.

$$
\begin{aligned}
n^2+n^2 &= 1^2 \\
2n^2 &= 1 \\
n^2 &= \frac{1}{2} \\
n &= \pm\sqrt{\frac{1}{2}} \\
n &= \pm\frac{1}{\sqrt{2}} \\
n &= \pm\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}
\end{aligned}
$$

Because the angle is in the first quadrant, the x and y coordinates are positive.

9. An angle in the first quadrant, as the tangent is the ratio of two positive numbers. And, angle in the third quadrant, as the tangent in the ratio of two negative numbers, which will be positive.
10. The terminal side of the angle is a reflection of the terminal side of 30°. From this, students should see that the ordered pair is $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.
11. Students should notice that tangent is the ratio of $\frac{\sin}{\cos}$, which is $\frac{y}{x}$, which is also slope.