3.1 Fundamental Identities

1. $\tan 270^\circ = \frac{\sin 270^\circ}{\cos 270^\circ} = \frac{-1}{0}$, you cannot divide by zero, therefore $\tan 270^\circ$ is undefined.
2. If $\cos\left(\frac{\pi}{2} - x\right) = \frac{4}{5}$, then, by the cofunction identities, $\sin x = \frac{4}{5}$. Because sine is odd, $\sin(-x) = -\frac{4}{5}$.
3. If $\tan(-x) = -\frac{5}{12}$, then $\tan x = \frac{5}{12}$. Because $\sin x = -\frac{5}{13}$, cosine is also negative, so $\cos x = -\frac{12}{13}$.
4. Use the reciprocal and cofunction identities to simplify

$$\sec x \cos\left(\frac{\pi}{2} - x\right)$$
$$\frac{1}{\cos x} \cdot \sin x$$
$$\frac{\sin x}{\cos x}$$
$$\tan x$$

5. (a) Using the sides 5, 12, and 13 and in the first quadrant, it doesn't really matter which is cosine or sine.
   - So, $\sin^2\theta + \cos^2\theta = 1$ becomes $\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = 1$. *Simplifying, we get: $\frac{25}{169} + \frac{144}{169} = 1$,
   - Finally $\frac{169}{169} = 1$.

   (b) $\sin^2\theta + \cos^2\theta = 1$ becomes $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$. Simplifying we get: $\frac{1}{4} + \frac{3}{4} = 1$ and $\frac{4}{4} = 1$.
6. To prove $\tan^2\theta + 1 = \sec^2\theta$, first use $\frac{\sin\theta}{\cos\theta} = \tan\theta$ and change $\sec^2\theta = \frac{1}{\cos^2\theta}$.

$$\tan^2\theta + 1 = \sec^2\theta$$
$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$
$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$
$$\sin^2\theta + \cos^2\theta = 1$$

7. If $\csc z = \frac{17}{8}$ and $\cos z = -\frac{15}{17}$, then $\sin z = \frac{8}{17}$ and $\tan z = -\frac{8}{15}$. Therefore $\cot z = -\frac{15}{8}$.
8. (a) Factor $\sin^2\theta - \cos^2\theta$ using the difference of squares.

$$\sin^2\theta - \cos^2\theta = (\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$$

   (b) $\sin^2\theta + 6\ \sin\theta + 8 = (\sin\theta + 4)(\sin\theta + 2)$
9. You will need to factor and use the $\sin^2\theta + \cos^2\theta = 1$ identity.

$$\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta - \cos^2\theta}$$
$$= \frac{(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)}{\sin^2\theta - \cos^2\theta}$$
$$= \sin^2\theta + \cos^2\theta$$
$$= 1$$

10. To rewrite $\frac{\cos x}{\sec x - 1}$ so it is only in terms of cosine, start with changing secant to cosine.

$$\frac{\cos x}{\sec x - 1} = \frac{\cos x}{\frac{1}{\cos x} - 1}$$ Now, simplify the denominator.
$$\frac{\cos x}{\frac{1}{\cos x} - 1} = \frac{\cos x}{\frac{1 - \cos x}{\cos x}}$$

Multiply by the reciprocal $\frac{\cos x}{\frac{1-\cos x}{\cos x}} = \cos x \div \frac{1-\cos x}{\cos x} = \cos x \cdot \frac{\cos x}{1-\cos x} = \frac{\cos^2 x}{1-\cos x}$

11. The easiest way to prove that tangent is odd to break it down, using the Quotient Identity.

$$\begin{aligned}
\tan(-x) &= \frac{\sin(-x)}{\cos(-x)} && \text{from this statement, we need to show that } \tan(-x) = -\tan x \\
&= \frac{-\sin x}{\cos x} && \text{because } \sin(-x) = -\sin x \text{ and } \cos(-x) = \cos x \\
&= -\tan x
\end{aligned}$$