3.2 Proving Identities

1. Step 1: Change everything into sine and cosine

$$\sin x \tan x + \cos x = \sec x$$
$$\sin x \cdot \frac{\sin x}{\cos x} + \cos x = \frac{1}{\cos x}$$

Step 2: Give everything a common denominator, $\cos x$.

$$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} = \frac{1}{\cos x}$$

Step 3: Because the denominators are all the same, we can eliminate them.

$$\sin^2 x + \cos^2 x = 1$$

We know this is true because it is the Trig Pythagorean Theorem

2. Step 1: Pull out a $\cos x$

$$\cos x - \cos x \sin^2 x = \cos^3 x$$
$$\cos x(1 - \sin^2 x) = \cos^3 x$$

Step 2: We know $\sin^2 x + \cos^2 x = 1$, so $\cos^2 x = 1 - \sin^2 x$ is also true, therefore $\cos x(\cos^2 x) = \cos^3 x$. This, of course, is true, we are done!

3. Step 1: Change everything in to sine and cosine and find a common denominator for left hand side.

$$\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$$
$$\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = \frac{2}{\sin x} \leftarrow \text{LCD}: \ \sin x(1+\cos x)$$
$$\frac{\sin^2 x + (1+\cos x)^2}{\sin x(1+\cos x)}$$

Step 2: Working with the left side, FOIL and simplify.

$$\frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)} \qquad \rightarrow \text{FOIL } (1+\cos x)^2$$
$$\frac{\sin^2 x + \cos^2 x + 1 + 2\cos x}{\sin x(1+\cos x)} \qquad \rightarrow \text{move } \cos^2 x$$
$$\frac{1 + 1 + 2\cos x}{\sin x(1+\cos x)} \qquad \rightarrow \sin^2 x + \cos^2 x = 1$$
$$\frac{2 + 2\cos x}{\sin x(1+\cos x)} \qquad \rightarrow \text{add}$$
$$\frac{2(1+\cos x)}{\sin x(1+\cos x)} \qquad \rightarrow \text{fator out 2}$$
$$\frac{2}{\sin x} \qquad \rightarrow \text{cancel } (1+\cos x)$$

4. Step 1: Cross-multiply

$$\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$$
$$\sin^2 x=(1+\cos x)(1-\cos x)$$

Step 2: Factor and simplify

$$\sin^2 x=1-\cos^2 x$$
$$\sin^2 x+\cos^2 x=1$$

5. Step 1: Work with left hand side, find common denominator, FOIL and simplify, using $\sin^2 x+\cos^2 x=1$.

$$\frac{1}{1+\cos x}+\frac{1}{1-\cos x}=2+2\cot^2 x$$
$$\frac{1-\cos x+1+\cos x}{(1+\cos x)(1-\cos x)}$$
$$\frac{2}{1-\cos^2 x}$$
$$\frac{2}{\sin^2 x}$$

Step 2: Work with the right hand side, to hopefully end up with $\frac{2}{\sin^2 x}$.

$$=2+2\cot^2 x$$
$$=2+2\,\frac{\cos^2 x}{\sin^2 x}$$
$$=2\left(1+\frac{\cos^2 x}{\sin^2 x}\right) \quad \rightarrow \text{factor out the 2}$$
$$=2\left(\frac{\sin^2 x+\cos^2 x}{\sin^2 x}\right) \quad \rightarrow \text{common denominator}$$
$$=2\left(\frac{1}{\sin^2 x}\right) \quad \rightarrow \text{trig pythagorean theorem}$$
$$=\frac{2}{\sin^2 x} \quad \rightarrow \text{simply/multiply}$$

Both sides match up, the identity is true.

6. Step 1: Factor left hand side

$$\cos^4 b-\sin^4 b \quad \Big| \quad 1-2\sin^2 b$$
$$(\cos^2 b+\sin^2 b)(\cos^2 b-\sin^2 b) \quad \Big| \quad 1-2\sin^2 b$$
$$\cos^2 b-\sin^2 b \quad \Big| \quad 1-2\sin^2 b$$

Step 2: Substitute $1-\sin^2 b$ for $\cos^2 b$ because $\sin^2 x+\cos^2 x=1$.

$$(1-\sin^2 b)-\sin^2 b \quad \Big| \quad 1-2\sin^2 b$$
$$1-\sin^2 b-\sin^2 b \quad \Big| \quad 1-2\sin^2 b$$
$$1-2\ \sin^2 b \quad \Big| \quad 1-2\sin^2 b$$

7. Step 1: Find a common denominator for the left hand side and change right side in terms of sine and cosine.

$$\frac{\sin y+\cos y}{\sin y}-\frac{\cos y-\sin y}{\cos y}=\sec y\csc y$$
$$\frac{\cos y(\sin y+\cos y)-\sin y(\cos y-\sin y)}{\sin y\cos y}=\frac{1}{\sin y\cos y}$$

Step 2: Work with left side, simplify and distribute.

$$\frac{\sin y\cos y+\cos^2 y-\sin y\cos y+\sin^2 y}{\sin y\cos y}$$
$$\frac{\cos^2 y+\sin^2 y}{\sin y\cos y}$$
$$\frac{1}{\sin y\cos y}$$

8. Step 1: Work with left side, change everything into terms of sine and cosine.

$$(\sec x-\tan x)^2=\frac{1-\sin x}{1+\sin x}$$
$$\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2$$
$$\left(\frac{1-\sin x}{\cos x}\right)^2$$
$$\frac{(1-\sin x)^2}{\cos^2 x}$$

Step 2: Substitute $1-\sin^2 x$ for $\cos^2 x$ because $\sin^2 x+\cos^2 x=1$

$$\frac{(1-\sin x)^2}{1-\sin^2 x}\rightarrow \text{be careful, these are NOT the same!}$$

Step 3: Factor the denominator and cancel out like terms.

$$\frac{(1-\sin x)^2}{(1+\sin x)(1-\sin x)}$$
$$\frac{1-\sin x}{1+\sin x}$$

9. Plug in $\frac{5\pi}{6}$ for x into the formula and simplify.

$$2\sin x\cos x=\sin 2x$$
$$2\sin\frac{5\pi}{6}\cos\frac{5\pi}{6}=\sin 2\cdot\frac{5\pi}{6}$$
$$2\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)=\sin\frac{5\pi}{3}$$

This is true because $\sin 300^\circ$ is $-\frac{\sqrt{3}}{2}$

10. Change everything into terms of sine and cosine and simplify.

$$\sec x\cot x=\csc x$$
$$\frac{1}{\cos x}\cdot\frac{\cos x}{\sin x}=\frac{1}{\sin x}$$
$$\frac{1}{\sin x}=\frac{1}{\sin x}$$