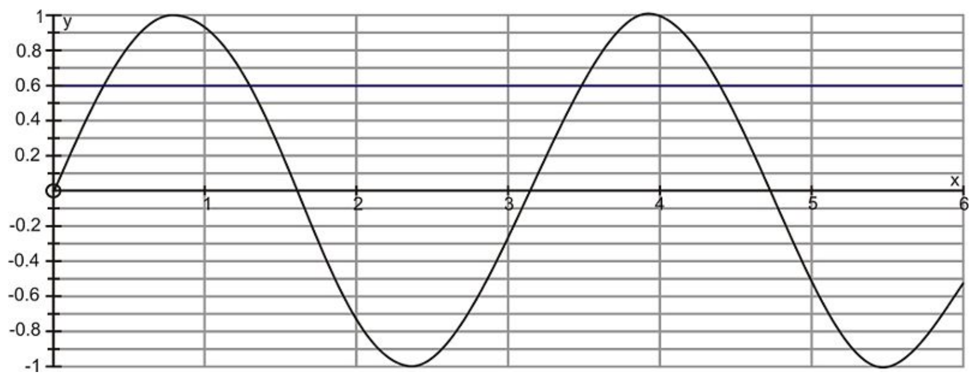
3.3 Solving Trigonometric Equations

1. Answer:

- Because the problem deals with 2θ, the domain values must be doubled, making the domain $0 \le 2\theta < 4\pi$
- The reference angle is $\alpha = \sin^{-1} 0.6 = 0.6435$
- $2\theta = 0.6435, \pi - 0.6435, 2\pi + 0.6435, 3\pi - 0.6435$
- $2\theta = 0.6435, 2.4980, 6.9266, 8.7812$
- The values for θ are needed so the above values must be divided by 2.
- $\theta = 0.3218, 1.2490, 3.4633, 4.3906$
- The results can readily be checked by graphing the function. The four results are reasonable since they are the only results indicated on the graph that satisfy $\sin 2\theta = 0.6$.



2.

$$\cos^2 x = \frac{1}{16}$$
$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{16}}$$
$$\cos x = \pm \frac{1}{4}$$

Then $\cos x = \frac{1}{4}$ or $\cos x = -\frac{1}{4}$

$$\cos^{-1} \frac{1}{4} = x \qquad \cos^{-1} -\frac{1}{4} = x$$
$$x = 1.3181 \textit{ radians} \qquad x = 1.8235 \textit{ radians}$$

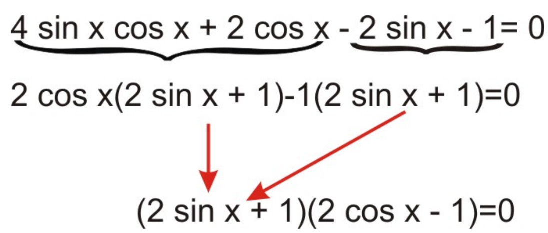
- However, $\cos x$ is also positive in the fourth quadrant, so the other possible solution for $\cos x = \frac{1}{4}$ is $2\pi - 1.3181 = 4.9651$ *radians*
- $\cos x$ is also negative in the third quadrant, so the other possible solution for $\cos x = -\frac{1}{4}$ is $2\pi - 1.8235 = 4.4597$ *radians*

3.

$$\tan^2 x = 1$$
$$\tan x = \pm\sqrt{1}$$
$$\tan x = \pm 1$$

- So, $\tan x = 1$ or $\tan x = -1$.
- Therefore, x is all critical values corresponding with $\frac{\pi}{4}$ within the interval. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

4. Use factoring by grouping.



$$\begin{aligned} 2\sin x + 1 &= 0 \quad \text{or} & 2\cos x - 1 &= 0 \\ 2\sin x &= -1 & 2\cos x &= 1 \\ \sin x &= -\frac{1}{2} & \cos x &= \frac{1}{2} \\ x &= \frac{7\pi}{6}, \frac{11\pi}{6} & x &= \frac{\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

5. You can factor this one like a quadratic.

$$\begin{aligned} \sin^2 x - 2\sin x - 3 &= 0 \\ (\sin x - 3)(\sin x + 1) &= 0 \\ \sin x - 3 = 0 & & \sin x + 1 = 0 \\ \sin x = 3 & \quad \text{or} & \sin x = -1 \\ x = \sin^{-1}(3) & & x = \frac{3\pi}{2} \end{aligned}$$

For this problem the only solution is $\frac{3\pi}{2}$ because sine cannot be 3 (it is not in the range).

6.

$$\begin{aligned} \tan^2 x &= 3\tan x \\ \tan^2 x - 3\tan x &= 0 \\ \tan x(\tan x - 3) &= 0 \\ \tan x &= 0 \quad \text{or} \quad \tan x = 3 \\ x &= 0, \pi \quad\quad x = 1.25 \end{aligned}$$

7. $2\sin^2\frac{x}{4}-3\cos\frac{x}{4}=0$

$$\begin{aligned}
2\left(1-\cos^2\frac{x}{4}\right)-3\cos\frac{x}{4}&=0\\
2-2\cos^2\frac{x}{4}-3\cos\frac{x}{4}&=0\\
2\cos^2\frac{x}{4}+3\cos\frac{x}{4}-2&=0\\
\left(2\cos\frac{x}{4}-1\right)\left(\cos\frac{x}{4}+2\right)&=0\\
\swarrow \qquad\qquad \searrow&\\
2\cos\frac{x}{4}-1=0 \quad \text{or} \quad \cos\frac{x}{4}+2&=0\\
2\cos\frac{x}{4}=1 \qquad\qquad \cos\frac{x}{4}&=-2\\
\cos\frac{x}{4}&=\frac{1}{2}\\
\frac{x}{4}=\frac{\pi}{3} \quad \text{or} \quad &\frac{5\pi}{3}\\
x=\frac{4\pi}{3} \quad \text{or} \quad &\frac{20\pi}{3}
\end{aligned}$$

$\frac{20\pi}{3}$ is eliminated as a solution because it is outside of the range and $\cos\frac{x}{4}=-2$ will not generate any solutions because -2 is outside of the range of cosine. Therefore, the only solution is $\frac{4\pi}{3}$.

8.

$$\begin{aligned}
3-3\sin^2 x&=8\sin x\\
3-3\sin^2 x-8\sin x&=0\\
3\sin^2 x+8\sin x-3&=0\\
(3\sin x-1)(\sin x+3)&=0\\
3\sin x-1=0 \quad \text{or} \quad \sin x+3&=0\\
3\sin x&=1\\
\sin x=\frac{1}{3} \qquad \sin x&=-3\\
x=0.3398\ \textit{radians} \quad &\text{No solution exists}\\
x=\pi-0.3398=2.8018\ \textit{radians}&
\end{aligned}$$

9. $2\sin x\tan x=\tan x+\sec x$

$$\begin{aligned}
2\sin x\cdot\frac{\sin x}{\cos x}&=\frac{\sin x}{\cos x}+\frac{1}{\cos x}\\
\frac{2\sin^2 x}{\cos x}&=\frac{\sin x+1}{\cos x}\\
2\sin^2 x&=\sin x+1\\
2\sin^2 x-\sin x-1&=0\\
(2\sin x+1)(\sin x-1)&=0\\
2\sin x+1=0 \quad \text{or} \quad \sin x-1&=0\\
2\sin x=-1 \qquad \sin x&=1\\
\sin x&=-\frac{1}{2}
\end{aligned}$$

$$x=\frac{7\pi}{6},\frac{11\pi}{6}$$

One of the solutions is not $\frac{\pi}{2}$, because $\tan x$ and $\sec x$ in the original equation are undefined for this value of x.

10.

$$\begin{aligned}
2\cos^2 x + 3\sin x - 3 &= 0 \\
2(1-\sin^2 x) + 3\sin x - 3 &= 0 \text{ Pythagorean Identity} \\
2 - 2\sin^2 x + 3\sin x - 3 &= 0 \\
-2\sin^2 x + 3\sin x - 1 &= 0 \text{ Multiply by } -1 \\
2\sin^2 x - 3\sin x + 1 &= 0 \\
(2\sin x - 1)(\sin x - 1) &= 0 \\
2\sin x - 1 = 0 \quad &\text{or} \quad \sin x - 1 = 0 \\
2\sin x &= 1 \\
\sin x = \frac{1}{2} \quad & \quad \sin x = 1 \\
x = \frac{\pi}{6}, \frac{5\pi}{6} \quad & \quad x = \frac{\pi}{2}
\end{aligned}$$

11. $\tan^2 x + \tan x - 2 = 0$

$$\begin{aligned}
\frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2} &= \tan x \\
\frac{-1 \pm \sqrt{1+8}}{2} &= \tan x \\
\frac{-1 \pm 3}{2} &= \tan x \\
\tan x = -2 \quad &\text{or} \quad 1
\end{aligned}$$

$\tan x = 1$ when $x = \frac{\pi}{4}$, in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\tan x = -2$ when $x = -1.107$ *rad*

12. $5\cos^2\theta - 6\sin\theta = 0$ over the interval $[0, 2\pi]$.

$$\begin{aligned}
5\left(1-\sin^2 x\right) - 6\sin x &= 0 \\
-5\sin^2 x - 6\sin x + 5 &= 0 \\
5\sin^2 x + 6\sin x - 5 &= 0 \\
\frac{-6 \pm \sqrt{6^2 - 4(5)(-5)}}{2(5)} &= \sin x \\
\frac{-6 \pm \sqrt{36+100}}{10} &= \sin x \\
\frac{-6 \pm \sqrt{136}}{10} &= \sin x \\
\frac{-6 \pm 2\sqrt{34}}{10} &= \sin x \\
\frac{-3 \pm \sqrt{34}}{5} &= \sin x
\end{aligned}$$

$x = \sin^{-1}\left(\frac{-3+\sqrt{34}}{5}\right)$ or $\sin^{-1}\left(\frac{-3-\sqrt{34}}{5}\right)$ $x = 0.6018$ *rad* or 2.5398 *rad* from the first expression, the second expression will not yield any answers because it is out the the range of sine.