

## 5.2 Area of a Triangle

1. Answers:

- $A = \frac{1}{2}bh$
- Heron's formula
- $K = \frac{1}{2}bc \sin A$
- $A = \frac{1}{2}bh$

2. Answers:

- $A = 22$
- $A = 14.3$
- $A = 2514.2$
- $A = 144.7$

3. Answers:

- $A = \frac{1}{2}bh$
- $K = \frac{1}{2}bc \sin A$
- $A = \frac{1}{2}bh$

4. Answers:

- $h = 89.2$
- $\angle A = 67^\circ$
- Area of  $\triangle ABC = 83.0$

5. Answers: (a) Use Heron's Formula, then multiply your answer by 4, for the 4 sides.

- $s = \frac{1}{2}(375 + 375 + 590) = 670$
- $A = \sqrt{670(670 - 375)(670 - 375)(670 - 590)} = 68,297.4$
- 68,297.4 multiplied by 4 = 273,189.8 total square feet.

(b)  $\frac{273,189.8}{25} \approx 10,928$  gallons of paint are needed.

6. Using Heron's Formula,  $s$  and calculate the area:

- $s = \frac{1}{2}(8.2 + 14.6 + 16.3) = 19.55$
- $A = \sqrt{19.55(19.55 - 8.2)(19.55 - 14.6)(19.55 - 16.3)} = 59.75 \text{ sq. ft.}$
- He will need 2 bundles ( $\frac{59.75}{33.3} = 1.8$ ).
- The shingles will cost him  $2 * \$15.45 = \$30.90$
- 6.92 square feet will go to waste ( $66.67 - 59.75 = 6.92$ ).

7. Answers:

- Use  $K = \frac{1}{2}bc \sin A$ ,  $K = \frac{1}{2}(186)(205) \sin 148^\circ$ . So, the area that needs to be replaced is 10102.9 square yards.
- $K = \frac{1}{2}(186)(288) \sin 148^\circ = 14193.4$ , the area has increased by 4090.5 yards.

8. You need to use the  $K = \frac{1}{2}bc \sin A$  formula to find  $DE$  and  $GF$ .

$$56.5 = \frac{1}{2}(13.6)DE \sin 39^\circ \rightarrow DE = 13.2 \qquad 84.7 = \frac{1}{2}(13.6)EF \sin 60^\circ \rightarrow EF = 14.4$$

Second, you need to find sides  $DG$  and  $GF$  using the Law of Cosines.

$$DG^2 = 13.2^2 + 13.6^2 - 2 \cdot 13.2 \cdot 13.6 \cdot \cos 39^\circ \rightarrow DG = 8.95$$

$$GF^2 = 14.4^2 + 13.6^2 - 2 \cdot 14.4 \cdot 13.6 \cdot \cos 60^\circ \rightarrow GF = 14.0$$

The perimeter of the quadrilateral is 50.55.

9. Answer:

- First, find  $BD$  by using the Pythagorean Theorem.  $BD = \sqrt{16.2^2 - 14.4^2} = 7.42$ .
- Then, using the area and formula ( $A = \frac{1}{2}bh$ ), you can find  $AC$ .  $232.96 = \frac{1}{2}(7.42)AC$
- $AC = 62.78$ .
- $DC = 62.78 - 14.4 = 48.38$ .

10. Answer:

$$d^2 = e^2 + f^2 - 2ef \cos D$$

$$e^2 = d^2 + f^2 - 2df \cos E$$

$$f^2 = d^2 + e^2 - 2de \cos F$$

All three versions of the Law of Cosines

Add the three formulas together, we get:

$$d^2 + e^2 + f^2 = e^2 + f^2 - 2ef \cos D + d^2 + f^2 - 2df \cos E + d^2 + e^2 - 2de \cos F$$

$$d^2 + e^2 + f^2 = 2(d^2 + e^2 + f^2) - 2(ef \cos D + df \cos E + de \cos F)$$

$$-(d^2 + e^2 + f^2) = -2(ef \cos D + df \cos E + de \cos F)$$

$$d^2 + e^2 + f^2 = 2(ef \cos D + df \cos E + de \cos F)$$