6.3 Converting Between Systems

1. For $A, r = -4$ and $\theta = \frac{5\pi}{4}$

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$x = -4\cos\frac{5\pi}{4} \qquad y = -4\sin\frac{5\pi}{4}$$
$$x = -4\left(-\frac{\sqrt{2}}{2}\right) \qquad y = -4\left(-\frac{\sqrt{2}}{2}\right)$$
$$x = 2\sqrt{2} \qquad y = 2\sqrt{2}$$

For $B, r = -3$ and $\theta = 135^\circ$

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$x = -3\cos 135^\circ \qquad y = -3\sin 135^\circ$$
$$x = -3 - \frac{\sqrt{2}}{2} \qquad y = -3\frac{\sqrt{2}}{2}$$
$$x = \frac{3\sqrt{2}}{2} \qquad y = \frac{-3\sqrt{2}}{2}$$

For C, $r = 5$ and $\theta = \left(\frac{2\pi}{3}\right)$

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$x = 5\cos\frac{2\pi}{3} \qquad y = 5\sin\frac{2\pi}{3}$$
$$x = 5\left(-\frac{1}{2}\right) \qquad y = 5\left(\frac{\sqrt{3}}{2}\right)$$
$$x = -2.5 \qquad y = \frac{5\sqrt{3}}{2}$$

2. Answers:

a.

$$r = 6\cos\theta$$
$$r^2 = 6r\cos\theta$$
$$x^2 + y^2 = 6x$$
$$x^2 - 6x + y^2 = 0$$
$$x^2 - 6x + 9 + y^2 = 9$$
$$(x-3)^2 + y^2 = 9$$

b.

$$r\sin\theta = -3$$
$$y = -3$$

c.

$$
\begin{aligned}
r &= 2\sin\theta \\
r^2 &= 2r\sin\theta \\
x^2+y^2 &= 2y \\
y^2-2y &= -x^2 \\
y^2-2y+1 &= -x^2+1 \\
(y-1)^2 &= -x^2+1 \\
x^2+(y-1)^2 &= 1
\end{aligned}
$$

d.

$$
\begin{aligned}
r\sin^2\theta &= 3\cos\theta \\
r^2\sin^2\theta &= 3r\cos\theta \\
y^2 &= 3x
\end{aligned}
$$

3. Answers:
   a. For $A(-2,5) x=-2$ and $y=5$. The point is located in the second quadrant and $x<0$.

   $$r=\sqrt{(-2)^2+(5)^2}=\sqrt{29}\approx 5.39,\ \theta = Arc\ \tan\frac{5}{-2}+\pi = 1.95.$$

   The polar coordinates for the rectangular coordinates $A(-2,5)$ are $A(5.39,1.95)$
   b. For $B(5,-4) x=5$ and $y=-4$. The point is located in the fourth quadrant and $x>0$.

   $$r=\sqrt{(5)^2+(-4)^2}=\sqrt{41}\approx 6.4,\ \theta=\tan^{-1}\left(\frac{-4}{5}\right)\approx -0.67$$

   The polar coordinates for the rectangular coordinates $B(5,-4)$ are $A(6.40,-0.67)$
   c. $C(1,9)$ is located in the first quadrant.

   $$r=\sqrt{1^2+9^2}=\sqrt{82}\approx 9.06,\ \theta=\tan^{-1}\frac{9}{1}\approx 83.66^\circ.$$

   d. $D(-12,-5)$ is located in the third quadrant and $x<0$.

   $$r=\sqrt{(-12)^2+(-5)^2}=\sqrt{169}=13,\ \theta=\tan^{-1}\frac{5}{12}+180^\circ\approx 202.6^\circ.$$

4. Answers:
   a.

$$
\begin{aligned}
(x-4)^2+(y-3)^2 &= 25 \\
x^2-8x+16+y^2-6y+9 &= 25 \\
x^2-8x+y^2-6y+25 &= 25 \\
x^2-8x+y^2-6y &= 0 \\
x^2+y^2-8x-6y &= 0 \\
r^2-8(r\cos\theta)-6(r\sin\theta) &= 0 \\
r^2-8r\cos\theta-6r\sin\theta &= 0 \\
r(r-8\cos\theta-6\sin\theta) &= 0 \\
r=0 \text{ or } r-8\cos\theta-6\sin\theta &= 0 \\
r=0 \text{ or } r &= 8\cos\theta+6\sin\theta
\end{aligned}
$$

From graphing $r-8\cos\theta-6\sin\theta=0,$ we see that the additional solutions are 0 and 8.

b.

$$\begin{aligned}
3x - 2y &= 1 \\
3r\cos\theta - 2r\sin\theta &= 1 \\
r(3\cos\theta - 2\sin\theta) &= 1 \\
r &= \frac{1}{3\cos\theta - 2\sin\theta}
\end{aligned}$$

c.

$$\begin{aligned}
x^2 + y^2 - 4x + 2y &= 0 \\
r^2\cos^2\theta + r^2\sin^2\theta - 4r\cos\theta + 2r\sin\theta &= 0 \\
r^2(\sin^2\theta + \cos^2\theta) - 4r\cos\theta + 2r\sin\theta &= 0 \\
r(r - 4\cos\theta + 2\sin\theta) &= 0 \\
r = 0 \text{ or } r - 4\cos\theta + 2\sin\theta &= 0 \\
r = 0 \text{ or } r &= 4\cos\theta - 2\sin\theta
\end{aligned}$$

d.

$$\begin{aligned}
x^3 &= 4y^2 \\
(r\cos\theta)^3 &= 4(r\sin\theta)^2 \\
r^3\cos^3\theta &= 4r^2\sin^2\theta \\
\frac{4r^2\sin^2\theta}{r^3\cos^3\theta} &= 1 \\
\frac{4\tan^2\theta\sec\theta}{r} &= 1 \\
4\tan^2\theta\sec\theta &= r
\end{aligned}$$