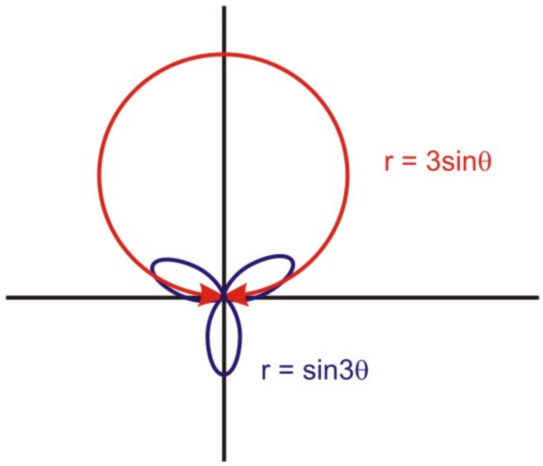
6.4 More with Polar Curves

1. Answer:

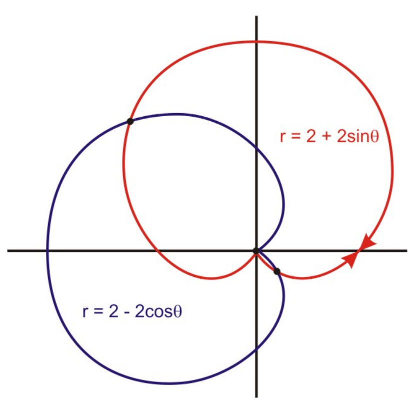


There appears to be one point of intersection.

$$\begin{aligned} r &= \sin 3\,\theta \\ 0 &= \sin 3\,\theta \\ 0 &= \theta \end{aligned} \qquad \begin{aligned} r &= 3\sin\theta \\ 0 &= 3\sin\theta \\ 0 &= \sin\theta \\ 0 &= \theta \end{aligned}$$

The point of intersection is (0, 0)

2. Answer:



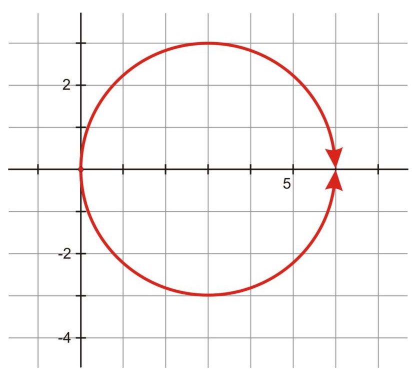
$$r = 2 + 2\sin\theta \qquad r = 2 + 2\sin\theta$$
$$r = 2 + 2\sin\left(\frac{3\pi}{4}\right) \qquad r = 2 + 2\sin\frac{7\pi}{4}$$
$$r \approx 3.4 \qquad r \approx 0.59$$
$$r = 2 + 2\sin\theta \qquad r = 2 - 2\cos\theta$$
$$0 = 2 + 2\sin\theta \qquad 0 = 2 - 2\cos\theta$$
$$-1 = \sin\theta \qquad 1 = \cos\theta$$
$$\theta = \frac{3\pi}{2} \qquad \theta = 0$$

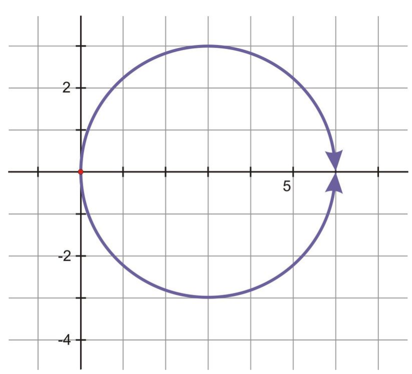
Since both equations have a solution at $r = 0$, that is (0, $\frac{3\pi}{2}$) and (0, 0), respectively, and these two points are equivalent, the two equations will intersect at (0, 0).

$$\begin{aligned} r &= 2 + 2\sin\theta \\ r &= 2 - 2\cos\theta \\ 2 + 2\sin\theta &= 2 - 2\cos\theta \\ 2\sin\theta &= -2\cos\theta \\ \frac{2\sin\theta}{2\cos\theta} &= -\frac{2\cos\theta}{2\cos\theta} \\ \frac{\sin\theta}{\cos\theta} &= -1 \\ \tan\theta &= -1 \\ \theta &= \frac{3\pi}{4} \text{ and } \theta = \frac{7\pi}{4} \end{aligned}$$

The points of intersection are $\left(3.4, \frac{3\pi}{4}\right), \left(0.59, \frac{7\pi}{4}\right)$ and $(0,0)$.

3. Answer:

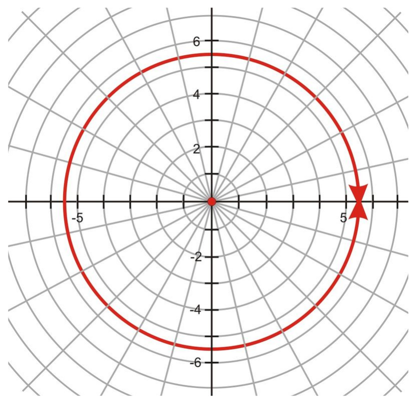




$$x^2 + y^2 = 6x$$
$$r^2 = 6(r\cos\theta) \qquad r^2 = x^2 + y^2 \quad \text{and} \quad x = y\cos\theta$$
$$r = 6\cos\theta \qquad \text{divide by } r$$

Both equations produced a circle with center $(3,0)$ and a radius of 3.

4. $r = -2 + \sin\theta$ and $r = 2 - \sin\theta$ are not equivalent because the sine has the opposite sign. $r = -2 + \sin\theta$ will be primarily above the horizontal axis and $r = 2 - \sin\theta$ will be mostly below. However, the two do have the same pole axis intercepts.
5. $r = -3 + 4\cos(-\pi)$ and $r = 3 + 4\cos\pi$ are equivalent because the sign of a does not matter, nor does the sign of θ.
6. Yes, the equations produced the same graph so they are equivalent.



7. Students answers will vary, but they need to include that b must be the same sign. They should also mention that the sign of a does not matter, nor does the sign of θ.
8. There are several answers here. The most obvious are any two pairs of circles, for example $r = 3$ and $r = 9$.