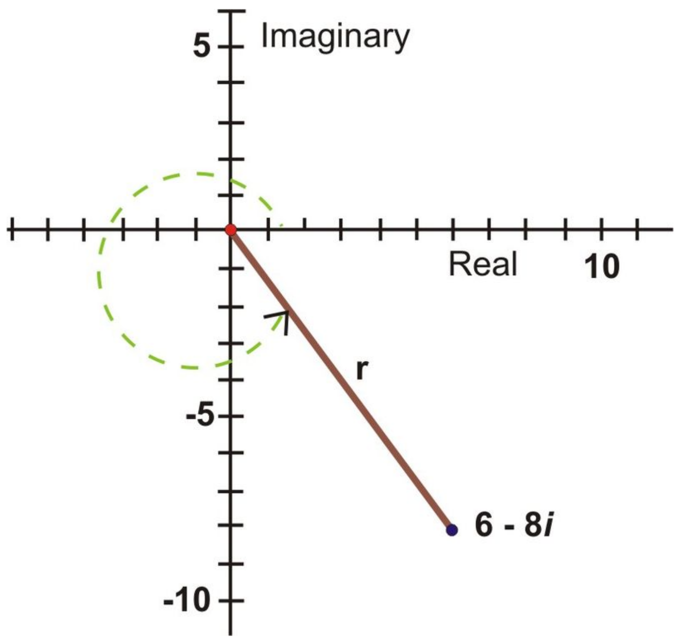
6.5 The Trigonometric Form of Complex Numbers

1. Answers:

    a. $5\ cis\frac{\pi}{6} = 5\angle\frac{\pi}{6} = 5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
    b. $3\angle 135^\circ = 3cis135^\circ = 3(\cos 135^\circ + i\sin 135^\circ)$
    c. $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2cis\frac{2\pi}{3} = 2\angle\frac{2\pi}{3}$

2. $6-8i$



$$6-8i$$

$$x = 6 \text{ and } y = -8 \qquad \tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2+y^2} \qquad \tan\theta = \frac{-8}{6}$$

$$r = \sqrt{(6)^2+(-8)^2} \qquad \theta = -53.1^\circ$$

$$r = 10$$

Since θ is in the fourth quadrant then $\theta = -53.1^\circ + 360^\circ = 306.9^\circ$ Expressed in polar form $6-8i$ is $10(\cos 306.9^\circ + i\sin 306.9^\circ)$ or $10\angle 306.9^\circ$

3. Answers:

    a. $4+3i \rightarrow x = 4, y = 3$

$$r = \sqrt{4^2+3^2} = 5, \tan\theta = \frac{3}{4} \rightarrow \theta = 36.87^\circ \rightarrow 5(\cos 36.87^\circ + i\sin 36.87^\circ)$$

    b. $-2+9i \rightarrow x = -2, y = 9$

$$r = \sqrt{(-2)^2+9^2} = \sqrt{85} \approx 9.22, \tan\theta = -\frac{9}{2} \rightarrow \theta = 102.53^\circ \rightarrow 9.22(\cos 102.53^\circ + i\sin 102.53^\circ)$$

c. $7 - i \rightarrow x = 7, y = -1$

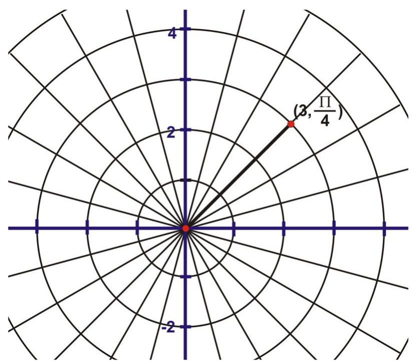
$$r = \sqrt{7^2 + 1^2} = \sqrt{50} \approx 7.07, \tan\theta = -\frac{1}{7} \rightarrow \theta = 351.87^\circ \rightarrow 7.07(\cos 351.87^\circ + i\sin 351.87^\circ)$$

d. $-5 - 2i \rightarrow x = -5, y = -2$

$$r = \sqrt{(-5)^2 + (-2)^2} = \sqrt{29} \approx 5.39, \tan\theta = \frac{2}{5} \rightarrow \theta = 201.8^\circ \rightarrow 5.39(\cos 201.8^\circ + i\sin 201.8^\circ)$$

- Note: The range of a graphing calculator's $\tan^{-1}$ function is limited to Quadrants I and IV, and for points located in the other quadrants, such as $-2+9i$ in part b (in Quadrant II), you must add 180° to get the correct angle θ for numbers given in polar form.

4. Answer:



$$3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$
$$r = 3$$
$$x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$y = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

The standard form of the polar complex number $3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ is $\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$.

5. Answers:

a. $2cis\frac{\pi}{2} \rightarrow x = \cos\frac{\pi}{2} = 0, y = \sin\frac{\pi}{2} = 1 \rightarrow 2(0) + 2(1i) = 2i$

b. $4\angle\frac{5\pi}{6} \rightarrow x = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}, y = \sin\frac{5\pi}{6} = \frac{1}{2} \rightarrow 4\left(-\frac{\sqrt{3}}{2}\right) + 4\left(i\frac{1}{2}\right) = -2\sqrt{3} + 2i$

c. $8\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) \rightarrow x = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}, y = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \rightarrow 8\left(\frac{1}{2}\right) + 8\left(-\frac{\sqrt{3}}{2}i\right) = 4 - 4i\sqrt{3}$