3.3 Solutions to Exercises

1 - 5 To find the C intercept, evaluate $c(t)$. To find the t-intercept, solve $C(t) = 0$.

1. (a) C intercept at (0, 48)
   (b) t intercepts at (4,0), (-1,0), (6,0)

3. (a) C intercept at (0,0)
   (b) t intercepts at (2,0), (-1,0), (0,0)

5. $C(t) = 2t^4 - 8t^3 + 6t^2 = 2t^2(t^2 - 4t + 3) = 2t^2(t-1)(t-3)$.

   (a) C intercept at (0,0)

   (b) t intercepts at (0,0), (3,0) (-1,0)

7. Zeros: $x \approx -1.65,\ x \approx 3.64,\ x \approx 5.$

9. (a) as $t \to \infty, h(t) \to \infty$.

   (b) as $t \to -\infty, h(t) \to -\infty$

For part a of problem 9, we see that as soon as t becomes greater than 5, the function $\mathrm{h(t)} = 3(\mathrm{t}-5)^3(\mathrm{t}-3)^3(\mathrm{t}-2)$ will increase positively as it approaches infinity, because as soon as t is greater than 5, the numbers within each parentheses will always be positive. In b, notice as t approaches -∞, any negative number cubed will stay negative. If you multiply first three terms: $[3 * (\mathrm{t}-5)^3 * (\mathrm{t}-3)^3]$, as t approaches -∞, it will always create a positive number. When you then multiply that by the final number: $(t-2)$, you will be multiplying a negative: $(t-2)$, by a positive: $[3 * (\mathrm{t}-5)^3 * (\mathrm{t}-3)^3]$, which will be a negative number.

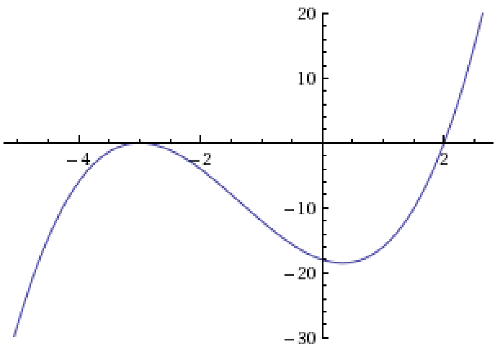
11. (a) as $t \to \infty, p(t) \to -\infty$

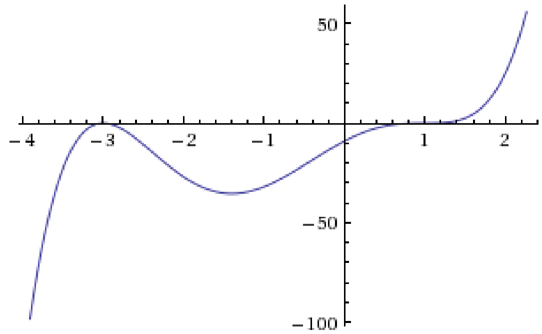
    (b) as $t \to -\infty, p(t) \to -\infty$

For part a of this problem as t approaches positive infinity, you will always have two parts of the equation $p(t) = -2t(t-1)(3-t)^2$, that are positive, once t is greater than 1: $[(t-1) * (3-t)^2]$, when multiplied together they stay positive. They are then multiplied by a number that will always be negative: -2t. A negative multiplied by a positive is always negative, so p(t) approaches -∞. For part b of this problem, as t approaches negative infinity, you will always have two parts of the equation that are always positive: $[-2t * (3-t)^2]$, when multiplied together stay positive. They are then multiplied by a number that will always be negative: (t-1). A negative multiplied by a positive is always negative, so p(t) approaches -∞.

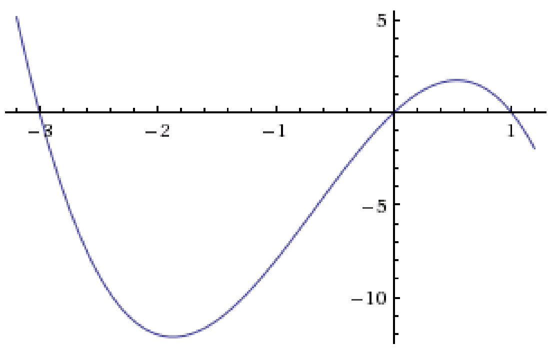
13. $f(x) = (x+3)^2(x-2)$

15. $h(x) = (x-1)^3(x+3)^2$





17. $m(x) = -2x(x-1)(x+3)$



19. $(x-3)(x-2)^2 > 0$ when $x > 3$

To solve the inequality $(x-3)(x-2)^2 > 0$, you first want to solve for x, when the function would be equal to zero. In this case, once you've solved for x, you know that when $f(x) = 0$, $x = 3$, and $x = 2$. You want to test numbers greater than, less than, and in-between these points, to see if these intervals are positive or negative. If an interval is positive it is part of your solution, and if it's negative it's not part of your solution. You test the intervals by plugging any number greater than 3, less than 2, or in between 2 and 3 into your inequality. For this problem, $(x-3)(x-2)^2 > 0$ is only positive when x is greater than 3. So your solution is: $(x-3)(x-2)^2 > 0$, when $x > 3$.

21. $(x-1)(x+2)(x-3) > 0$ when -2 < x < 1, and when $x > 3$

To solve the inequality$(x-1)(x+2)(x-3) > 0$, you first want to solve for x, when the function would be equal to zero. In this case, once you've solved for x, you know that when $f(x)=0,\ x=1,\ x=-2,$ and $x=3$. You want to test numbers greater than, less than, and in-between these points, to see if these intervals are positive or negative. If an interval is positive it is part of your solution, and if it's negative it's not part of your solution. You test the intervals by plugging any number greater than 3, less than -2, or in between -2 and 1, and in between 1 and 3 into your inequality. For this problem, $(x-1)(x+2)(x-3) > 0$ is positive when x is greater than 3, and when it's in between -2 and 1. So your solution is: $(x-1)(x+2)(x-3) > 0$ when $-2 < x < 1,$ and when $x > 3$.

23. The domain is the values of *x* for which the expression under the radical is nonnegative:

$-42+19x-2x^2 \geq 0$

$-(2x^2-19x+42) \geq 0$

$-(2x-7)(x-6) \geq 0$

Recall that this graph is a parabola which opens down, so the nonnegative portion is the interval between (and including) the *x*-intercepts: $\frac{7}{2} < x < 6$.

25. The domain is the values of *x* for which the expression under the radical is nonnegative:

$$4-5x-x^2 \geq 0$$

$$(x-4)(x-1) \geq 0$$

Recall that this graph is a parabola which opens up, so the nonnegative portions are the intervals outside of (and including) the *x*-intercepts: $x \leq 1$ and $x \geq 4$.

27. The domain is the values of *x* for which the expression under the radical is nonnegative, and since $(x+2)^2$ is always nonnegative, we need only consider where $x-3 > 0$, so the domain is $x \geq 3$.

29. The domain can be any numbers for which the denominator of $p(t)$ is nonzero, because you can't have a zero in the denominator of a fraction. So find what values of *t* make $t^2+2t-8=0$, and those values are not in the domain of $p(t)$. $t^2+2t-8=(t+4)(t-2)$, so the domain is ℝ where $x \neq -4\ and\ x \neq 2$.

31. $f(x) = -\frac{2}{3}(x+2)(x-1)(x-3)$

For problem 31, you can use the x intercepts you're given to get to the point $f(x) = a(x+2)(x-1)(x-3)$, because you know that if you solved for each of the x values you would end up with the horizontal intercepts given to you in the problem. Since your equation is of degree three, you don't need to raise any of your x values to a power, because if you foiled $(x+2)(x-1)(x-3)$ there will be an x^3, which is degree three. To solve for a, (your stretch factor, in this case $-\frac{2}{3}$) , you can plug the point your given, (in this case it's the y intercept $(0,-4)$) into your equation: $-4=(0+2)(0-1)(0-3)$, to solve for a.

33. $f(x)=\frac{1}{3}(x-3)^2(x-1)^2(x+3)$

For problem 33, you can use the x intercepts you're given to get to the point $f(x)=a(x-3)^2(x-1)^2(x+3)$, because you know that if you solved for each of the x values you would end up with the horizontal intercepts given to you in the problem. The problem tells you at what intercepts has what roots of multiplicity to give a degree of 5, which is why $(x-2)$ and $(x-1)$ are squared. To solve for a, (your stretch factor, in this case, $\frac{1}{3}$), you can plug the point your given, (in this case it's the y intercept (0,9)) into your equation: $9=(0-3)^2(0-1)^2(0+3)$, to solve for a.

35. $f(x)=-15(x-1)^2(x-3)^3$

For problem 35, you can use the x intercepts you're given to get to the point $f(x)=a(x-1)^2(x-3)^3$, because you know that if you solved for each of the x values you would end up with the horizontal intercepts given to you in the problem. The problem tells you at what intercepts has what roots of multiplicity to give a degree of 5, which is why $(x-1)$ is squared, and $(x-3)$ is cubed. To solve for a, (your stretch factor, in this case, -15), you can plug the point your given, (in this case it's (2,15)) into your equation: $15=(2-1)^2(2-3)^3$, to solve for a.

37. The *x*-intercepts of the graph are (-2, 0), (1, 0), and (3, 0). Then $f(x)$ must include the factors $(x+2)$, $(x-1)$, and $(x-3)$ to ensure that these points are on the graph of $f(x)$, and there cannot be any other factors since the graph has no other *x*-intercepts. The graph passes through these three *x*-intercepts without any flattening behavior, so they are single zeros. Filling in what we know so far about the function: $f(x)=a(x+2)(x-1)(x-3)$. To find the value of a, we can use the *y*-intercept, (0, 3):

$$3=a(0+2)(0-1)(0-3)$$

$$3=6a$$

$$a = \frac{1}{2}$$

Then we conclude that $f(x) = \frac{1}{2}(x+2)(x-1)(x-3)$.

39. $f(x) = -(x+1)^2(x-2)$

41. $f(x) = -\frac{1}{24}(x+3)(x+2)(x-2)(x-4)$

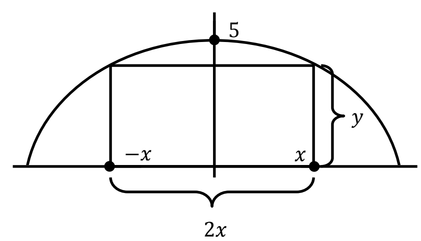
43. $f(x) = \frac{1}{24}(x+4)(x+2)(x-3)^2$

45. $f(x) = \frac{3}{32}(x+2)^2(x-3)^2$

47. $f(x) = \frac{1}{6}(x+3)(x+2)(x-1)^3$

49. $f(x) = -\frac{1}{16}(x+3)(x+1)(x-2)^2(x-4)$

51. See the diagram below. The area of the rectangle is $A = 2xy$, and $y = 5 - x^2$, so $A = 2x(5-x^2) = 10x - 2x^3$. Using technology, evaluate the maximum of $10x - 2x^3$. The y-value will be maximum area, and the x-value will be half of base length. Dividing the y-value by the x-value gives us the height of the rectangle. The maximum is at $x = 1.29, y = 8.61$. So, Base $= 2.58$, Height $= 6.67$.



problem 51