3C Problems – Adding vectors algebraically

Sample Problem
A hiker walks 25.5 km from her base camp at 35° south of east. On the second day, she walks 41.0 km in a direction 65° north of east, at which point she discovers a forest ranger's tower. Determine the magnitude and direction of her resultant displacement between the base camp and the ranger's tower.

S O L U T I O N
Select a coordinate system, draw a sketch of the vectors to be added, and label each vector. **Figure 3-13** depicts the situation drawn on a coordinate system. The positive *y*-axis points north and the positive *x*-axis points east. The origin of the axes is the base camp. In the chosen coordinate system, the hiker's direction q*1* during the first day is signified by a negative angle because clockwise movement from the positive *x*-axis is conventionally considered to be a negative angle.

Given: $\theta_1 = -35°$
$\theta_2 = 65°$
$d1 = 25.5$ km
$d2 = 41.0$ km

Unknown: d = ?
q = ?

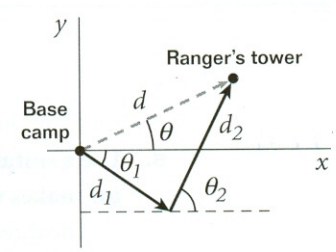

Figure 3-13

Find the x **and** y **components of all vectors.**
Make a separate sketch of the displacements for each day. The values for each of the displacement components can be determined by using the sine and cosine functions. Because the hiker's angle on the first day is negative, the *y* component of her displacement during that day is negative.
$\sin\theta = \Delta y / d$
$\cos\theta = \Delta x / d$

For day 1: $\Delta x_1 = d_1 (\cos\theta_1) = (25.5 \text{ km}) [\cos(-35°)]$
(Figure 3-14)
$\Delta x_1 = 21$ km
$\Delta y_1 = d1 (\sin\theta_1) = (25.5 \text{ km}) [\sin(-35°)]$
$\Delta y_1 = -15$ km

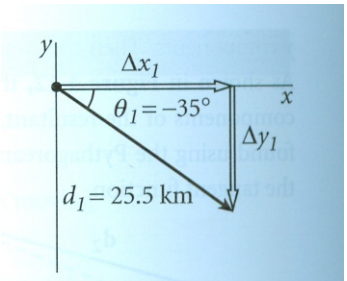

Figure 3-14

For day 2: $\Delta x_2 = d_2 (\cos\theta_2) = (41.0 \text{ km}) (\cos 65°)$
(Figure 3-15)
$\Delta x_2 = 17$ km
$\Delta y_2 = d_2 (\sin\theta_2) = (41.0 \text{ km}) (\sin 65°)$
$\Delta y_2 = 37$ km

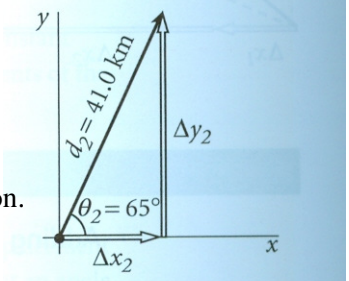

Figure 3-15

Find the x **and** y **components of the total displacement.**
First add together the *x* components to find the total displacement in the *x* direction. Then perform the same operation for the *y* direction.
$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 21 \text{ km} + 17 \text{ km} = 38 \text{ km}$
$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -15 \text{ km} + 37 \text{ km} = 22 \text{ km}$

Use the Pythagorean theorem to find the magnitude of the resultant vector.
Because the components $\Delta xtot$ and $\Delta ytot$ are perpendicular, the Pythagorean theorem can be used to find the magnitude of the resultant vector.
$d^2 = (\Delta x_{tot})^2 + (\Delta y_{tot})^2$
$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = (38 \text{ km})^2 + (22 \text{ km})^2$

Use a suitable trigonometric function to find the angle the resultant vector makes with the x-axis.

The direction of the resultant can be found using the tangent function.

$\theta = \tan^{-1} (\Delta y / \Delta x)$

$\theta = \tan^{-1} (\Delta y_{tot} / \Delta x_{tot})$

$\theta = \tan^{-1}(22 / 38)$

$\theta = 30\ ^\circ$ north of east

Evaluate your answer.

If the diagram is drawn to scale, compare the algebraic results with the drawing. The calculated magnitude seems reasonable because the distance from the base camp to the ranger's tower is longer than the distance hiked during the first day and slightly longer than the distance hiked during the second day. The calculated direction of the resultant seems reasonable because the angle in **Figure 3-13** looks to be about 30°.

3C Problems

1. A football player runs directly down the field for 35 m before turning to the right at an angle of 25° from his original direction and running an additional 15 m before getting tackled. What is the magnitude and direction of the runner's total displacement?

2. A plane travels 2.5 km at an angle of 35° to the ground, then changes direction and travels 5.2 km at an angle of 22° to the ground. What is the magnitude and direction of the plane's total displacement?

3. During a rodeo, a clown runs 8.0 m north, turns 35° east of north, and runs 3.5 m. Then, after waiting for the bull to come near, the clown turns due east and runs 5.0 m to exit the arena. What is the clown's total displacement?

4. An airplane flying parallel to the ground undergoes two consecutive displacements. The first is 75 km 30.0° west of north, and the second is 155 km 60.0° east of north. What is the total displacement of the airplane?