Linear Programming

Practical problems can be solved by linear programming. Linear programming is a procedure for finding the maximum or minimum value of a function in two variables, subject to given conditions on the variables called constraints. We can use linear programming problems to solve agriculture and manufacturing.

Steps to solving linear programming problems.

1. Read the problem carefully.
2. Write the constraints or inequalities.
3. Graph the inequalities. Find the feasible region.
4. Find the vertices of the feasible region.
5. Write a function to find the minimum or maximum value.
6. Plug the vertices into the function.
7. Find the maximum or minimum

Let's start with an example :

A sporting goods store manufacturer makes a $500 profit on a pool table and a $400 profit on an air hockey table. Department A requires 3 hours to make a pool table and 2 hours to make an air hockey table. Department B needs 2 hours to make a pool table and 3 hours to make an air hockey table. Department A has 500 hours available and department B has 450 hours available. How many pool tables and air hockey tables should be made to maximize profit?

Sometimes a chart can be very helpful in solving the problem:

Let A= number of air hockey tables produced
Let P= number of pool tables produced

	Pool tables	Air hockey tables	Total number of hours
Department A	**3 hrs**	**2 hrs**	**500**
Department B	**2 hrs**	**3 hrs**	**450**
Profit	**$500**	**$400**	

Let's write the constraints or inequalities:

$$P \geq 0$$

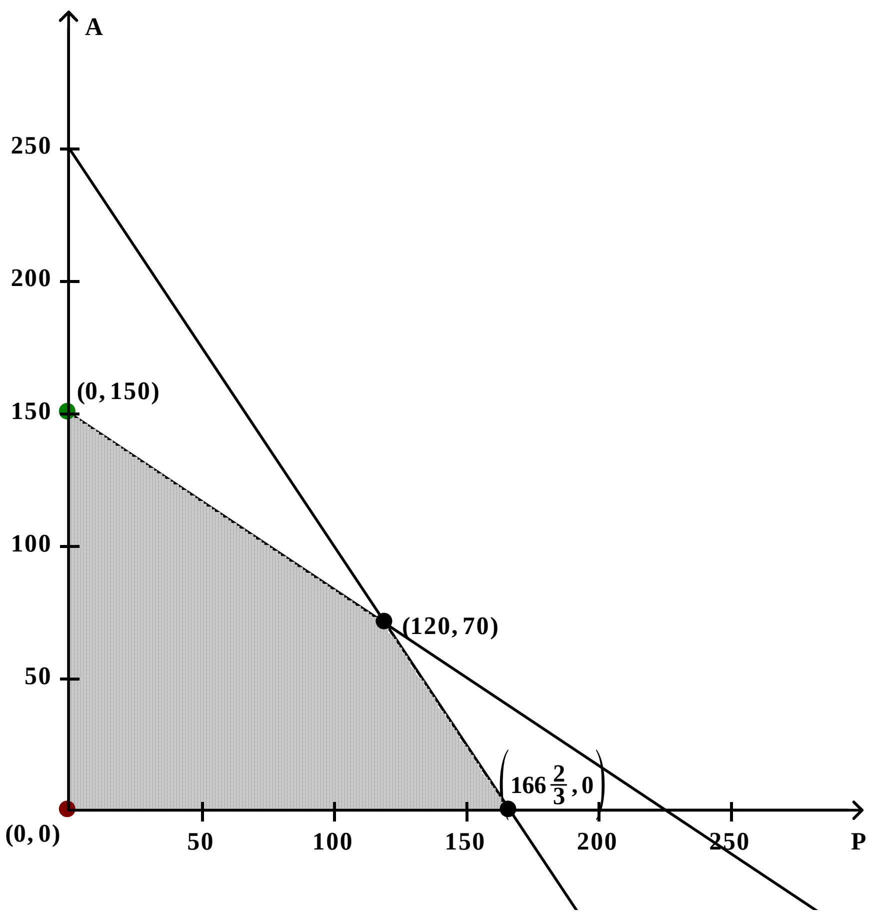
$$A \geq 0$$

These are always ***given*** constraints. Therefore the graph will always be in the first quadrant. Let's work on the other constraints:

$$3P + 2A \leq 500$$

$$2P + 3A \leq 450$$

Now we graph the 4 constraints and label the 4 vertices:



We now write the functions to maximize profit.

$$f(P, A) = 500P + 400A$$

Filling in the vertices into the maximized function we get

(P, A)	$P(P, A) = 500P + 400A$
$(0,0)$	$0
$(0,150)$	$60,000
$(120,70)$	$88,000
$(166,0)$	$83,000

Note: In the point $(166,0)$, we use 166 instead of $166\frac{2}{3}$ because $\frac{2}{3}$ of a table is not useful.

$\therefore$ 120 pool tables and 70 air hockey tables should be produced to yield a maximum profit of$88,000.