

LESSON 154 ANSWERS

1a. Total possible outcomes = $P(8, 8)$

Favorable outcomes = permutations of 7 remaining people after seating Emma in the first seat = $P(7, 7)$

$$\text{Probability} = \frac{P(7, 7)}{P(8, 8)} = \frac{7!}{8!} = \frac{1}{8}$$

1b. Favorable outcomes = permutations of 6 remaining people after seating Emma first and Brian last = $P(6, 6)$

$$\text{Probability} = \frac{P(6, 6)}{P(8, 8)} = \frac{6!}{8!} = \frac{1}{56}$$

1c. Favorable outcomes = Emma first and Brian last + Brian first and Emma last = $P(6, 6) + P(6, 6) = 2 \times P(6, 6)$

$$\text{Probability} = \frac{2 \cdot P(6, 6)}{P(8, 8)} = \frac{2 \cdot 6!}{8!} = \frac{1}{28}$$

1d. Emma and Brian must sit next to each other, so group them together and treat them as one person.

Favorable outcomes = permutations of 7 people with Emma-Brian or Brian-Emma = $2 \times P(7, 7)$

$$\text{Probability} = \frac{2 \cdot P(7, 7)}{P(8, 8)} = \frac{2 \cdot 7!}{8!} = \frac{1}{4}$$

2a. Total possible outcomes = $P(5, 5)$

Favorable outcomes = 1

$$\text{Probability} = \frac{1}{P(5, 5)} = \frac{1}{5!} = \frac{1}{120}$$

2b. Favorable outcomes = permutations of 4 remaining letters after placing A or E first = $2 \times P(4, 4)$

$$\text{Probability} = \frac{2 \cdot P(4, 4)}{P(5, 5)} = \frac{2 \cdot 4!}{5!} = \frac{2}{5}$$

2c. Favorable outcomes = A first and E last + E first and A last = $2 \times P(3, 3)$

$$\text{Probability} = \frac{2 \cdot P(3, 3)}{P(5, 5)} = \frac{2 \cdot 3!}{5!} = \frac{1}{10}$$

2d. The vowels should be placed second and fourth.

Favorable outcomes = A second and E fourth + E second and A fourth = $2 \times P(3, 3)$

$$\text{Probability} = \frac{2 \cdot P(3, 3)}{P(5, 5)} = \frac{2 \cdot 3!}{5!} = \frac{1}{10}$$

3a. Total possible outcomes = $C(12, 3)$

Favorable outcomes = combinations of choosing 3 balls from 3 red balls = $C(3, 3)$

$$\text{Probability} = \frac{C(3, 3)}{C(12, 3)} = \frac{1}{220}$$

3b. Favorable outcomes = combinations of choosing 3 balls from 5 pink balls = $C(5, 3)$

$$\text{Probability} = \frac{C(5, 3)}{C(12, 3)} = \frac{10}{220} = \frac{1}{22}$$

3c. Favorable outcomes = combinations of choosing 3 balls from 3 red and 5 pink balls = $C(8, 3)$

$$\text{Probability} = \frac{C(8, 3)}{C(12, 3)} = \frac{56}{220} = \frac{14}{55}$$

3d. Favorable outcomes = 3 ways to choose a red ball \times 5 ways to choose a pink ball \times 4 ways to choose a yellow ball = $3 \times 5 \times 4$

$$\text{Probability} = \frac{3 \cdot 5 \cdot 4}{C(12, 3)} = \frac{60}{220} = \frac{3}{11}$$

4a. Total possible outcomes = $C(52, 2)$

Favorable outcomes = combinations of choosing 2 cards from 13 clubs = $C(13, 2)$

$$\text{Probability} = \frac{C(13, 2)}{C(52, 2)} = \frac{78}{1326} = \frac{1}{17} = 5.9\%$$

4b. Favorable outcomes = combinations of choosing 2 cards from 39 non-clubs = $C(39, 2)$

$$\text{Probability} = \frac{C(39, 2)}{C(52, 2)} = \frac{741}{1326} = \frac{19}{34} = 55.9\%$$

4c. Favorable outcomes = 13 ways to choose a club \times 39 ways to choose a non-club = 13×39

$$\text{Probability} = \frac{13 \cdot 39}{C(52, 2)} = \frac{507}{1326} = \frac{13}{34} = 38.2\%$$

4d. $P(\text{at least 1 club}) = P(2 \text{ clubs}) + P(\text{exactly 1 club})$

$$\text{Probability} = \frac{1}{17} + \frac{13}{34} = \frac{15}{34} = 44.1\%$$

OR $P(\text{at least 1 club}) = 1 - P(\text{no clubs})$

$$\text{Probability} = 1 - \frac{19}{34} = \frac{15}{34} = 44.1\%$$