LESSON 133

1. perpendicular bisector, circumcenter

2. angle bisector, incenter

3. median, centroid

4. altitude, orthocenter

5. $\angle 1$ is larger than $\angle 2$ because 34 > 33.

6. $\angle 1$ is smaller than $\angle 2$ because 16 < 18.

7. Opposite angles are congruent, so $x = 62$.

Consecutive angles are supplementary, so $y = z = 180 - 62 = 118$.

8. The midsegment is half the sum of the bases, so $x = (7 + 9)/2 = 8$.

Base angles are congruent, so $y = 80$.

Non-base angles are supplementary, so $z = 180 - y = 100$.

9. Diagonals bisect opposite angles, so $m\angle 1 = 60°$.

Diagonals are perpendicular, so $m\angle 2 = 90°$.

Alternate interior angles are congruent, so $m\angle 3 = 60°$.

Diagonals bisect opposite angles, so $m\angle 4 = m\angle 3 = 60°$.

A triangle has 180°, so $m\angle 5 = 180 - 90 - m\angle 3 = 30°$.

There are many ways to find these angle measures that are all correct. For example, you could say $m\angle 1 = m\angle 2 = m\angle 3 = 60°$ because the four right triangles are congruent.

10. Non-vertex angles are congruent, so $m\angle 1 = 113°$.

Vertex angles are bisected by a diagonal, so $m\angle 2 = 27°$.

A triangle has 180°, so $m\angle 3 = 180 - 113 - m\angle 2 = 40°$.

A triangle has 180°, so $m\angle 4 = 180 - 27 - m\angle 1 = 40°$.

11. Statements (Reasons)

1. $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}$ (Given)
2. $\overline{AC} \cong \overline{CA}$ (Reflexive Property)
3. $\triangle ABC \cong \triangle CDA$ (SSS)
4. $\angle BAC \cong \angle DCA, \angle BCA \cong \angle DAC$ (CPCTC)
5. $\overline{BA} \parallel \overline{CD}, \overline{BC} \parallel \overline{AD}$ (If alternate interior angles are congruent, then lines are parallel.)
6. $ABCD$ is a parallelogram. (Def. of parallelogram)

12. Corresponding angles must be congruent, so both triangles have angles 51°, 39°, and $a°$.

A triangle has 180°, so $a = 180 - 51 - 39 = 90$.

13. Corresponding sides must be proportional.

$\frac{16}{16+8} = \frac{10}{a}$ → $16a = 10(16 + 8)$ → $a = 15$

$\frac{16}{16+8} = \frac{b}{b+7}$ → $16(b + 7) = b(16 + 8)$ → $b = 14$

14. Use the Altitude Rule [68.1].

$x^2 = 14(8)$; $x = 4\sqrt{7}$

15. Use the Triangle Side Splitter Theorem [69.1].

$\frac{20}{12} = \frac{15}{x}$ → $20x = 12(15)$ → $x = 9$

16. Use the Triangle Angle Bisector Theorem [70.1].

$\frac{4}{x} = \frac{15}{18}$ → $15x = 4(18)$ → $x = 24/5$

17. Use the Three Parallel Lines Theorem [69.2].

$\frac{12}{x} = \frac{16}{10}$ → $16x = 12(10)$ → $x = 15/2$

18. $AB = \sqrt{7^2 + 10^2} = \sqrt{149} \approx 12.2$

$m\angle A = \tan^{-1}(10/7) \approx 55°$

$m\angle B \approx 90 - 55 = 35°$

19. $AC = 14 \tan 24° \approx 6.2$

$AB = 14 / \cos 24° \approx 15.3$

$m\angle A = 90 - 24 = 66°$

20. area $= \frac{1}{2}(15)(19) \sin 40° \approx 91.6$

21. included angle = 180 – 103 = 77°

area = (6)(10) sin 77° ≈ 58.5

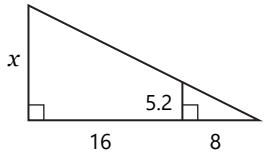
22 ~ 24. *Diagrams are not drawn to scale.*

22. $\frac{8}{8+16} = \frac{5.2}{x}$

$8x = (5.2)(8 + 16)$

$x = 15.6$

The street lamp is 15.6 feet tall.

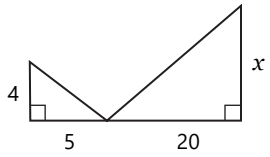


23. $\frac{4}{x} = \frac{5}{20}$

$5x = 4(20)$

$x = 16$

The tree is 16 feet tall.



24. $x = 90 / \tan 30° \approx 155.9$

$y = 90 / \tan 20° \approx 247.3$

$d = y - x \approx 91.4$

The distance between the two boats is about 91.4 ft.