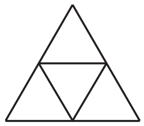
LESSON 135

1. triangular pyramid



2. $F = 4$, $E = 6$, $V = 4$
$4 + 4 = 6 + 2$

3 ~ 14. *All areas are in square units.*
All volumes are in cubic units.

3. $LA = Ph = (16 + 5 + 16 + 5)(7) = 294$
$SA = 2B + LA = 2(16)(5) + 294 = 454$
$V = Bh = 16(5)(7) = 560$

4. $SA = 2\pi r^2 + 2\pi rh = 2\pi(10)^2 + 2\pi(10)(6) = 320\pi$
$V = \pi r^2 h = \pi(10)^2(6) = 600\pi$

5. The dashed triangle has hypotenuse 17 and base 8. Use the Pythagorean Theorem to find $h = 15$.

$LA = \frac{1}{2}Pl = \frac{1}{2}(4)(16)(17) = 544$

$SA = B + LA = 16(16) + 544 = 800$

$V = \frac{1}{3}Bh = \frac{1}{3}(16)(16)(15) = 1{,}280$

6. Use the 5-12-13 Pythagorean triple to find $l = 13$.
$SA = \pi r^2 + \pi rl = \pi(5)^2 + \pi(5)(13) = 90\pi$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5)^2(12) = 100\pi$

7. $SA = 4\pi r^2 = 4\pi(6)^2 = 144\pi$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi$

8. SA = hemisphere LA + cylinder LA + hemisphere LA
= sphere SA + cylinder LA
$= 4\pi r^2 + 2\pi rh = 4\pi(3)^2 + 2\pi(3)(4) = 60\pi$

V = hemisphere V + cylinder V + hemisphere V
= sphere V + cylinder V

$= \frac{4}{3}\pi r^3 + \pi r^2 h = \frac{4}{3}\pi(3)^3 + \pi(3)^2(4) = 72\pi$

9. The volume ratio is 15/405 = 1/27, so the side ratio is 1/3 and the area ratio is 1/9. Set up and solve the proportion $1/9 = 9/x$ to get $x = 81$.

So, the base area of the larger pyramid is 81 ft^2.

10. The cross section is a square.

11. The solid of revolution is a cone.

12. balloon $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi$

$\approx 288(22/7) \approx 905.1$ in^3
time = volume / rate ≈ 905.1/5 ≈ 181
So, it will take about 181 minutes.

13. aquarium $V = Bh = 10(5)(5) = 250$
density = mass/volume
$62 = x/250$; $x = 15500$
So, the aquarium can hold 15,500 pounds of water.

14. population density = people/land area
$250 = x/82$; $x = 20500$
So, there are 20,500 people in the town.

15. length = distance between two endpoints
$= \sqrt{(1-3)^2 + (-4-2)^2} = 2\sqrt{10}$

16. midpoint $= \left(\frac{2+6}{2}, \frac{-9+7}{2}\right) = (4, -1)$

17. $AB = 17 - 2 = 15$
$P = A + 1/3$ of $AB = 2 + (1/3)(15) = 7$
So, P is at 7.

18. You need to move left and up to find P.
x-length of $\overline{AB} = 6 - 1 = 5$
x of P = x of A − (3/5)(x-length) = 6 − (3/5)(5) = 3

y-length of $\overline{AB} = 10 - (-5) = 15$
y of P = y of A + (3/5)(y-length) = −5 + (3/5)(15) = 4

So, P is at (3, 4).

19. original slope = 3
parallel slope = 3
point-slope form: $y - 1 = 3(x - 2)$
slope-intercept form: $y = 3x - 5$

20. original slope = 1/2
perpendicular slope = −2
point-slope form: $y - (-1) = (-2)(x - 1)$
slope-intercept form: $y = -2x + 1$

21. a. Find the line perpendicular to $y = x + 4$ passing through (0, −2).
original slope = 1
perpendicular slope = −1
y-intercept = −2
slope-intercept form: $y = -x - 2$

21. b. Find the Intersection between $y = x + 4$ and $y = -x - 2$.
$x + 4 = -x - 2$
$x = -3$
$y = -3 + 4 = 1$
The lines intersect at (−3, 1).

c. Find the distance between (0, −2) and (−3, 1).
$d = \sqrt{(-3-0)^2 + (1-(-2))^2} = 3\sqrt{2}$

22. $A(2, 1)$, $B(-2, 3)$, $C(0, -3)$

$AB = \sqrt{(-2-2)^2 + (3-1)^2} = 2\sqrt{5}$

$BC = \sqrt{(0-(-2))^2 + (-3-3)^2} = 2\sqrt{10}$

$AC = \sqrt{(0-2)^2 + (-3-1)^2} = 2\sqrt{5}$

$AB = AC$, so the triangle is isosceles.

23. $A(5, 0)$, $B(3, -3)$, $C(-2, 0)$, $D(0, 3)$

You can tell from the graph that it is not a rhombus. It looks like a parallelogram or a rectangle. Check the slopes of the sides. By the slope formula,

slope of $\overline{AB} = \dfrac{-3-0}{3-5} = \dfrac{3}{2}$

slope of $\overline{BC} = \dfrac{0-(-3)}{-2-3} = -\dfrac{3}{5}$

slope of $\overline{CD} = \dfrac{3-0}{0-(-2)} = \dfrac{3}{2}$

slope of $\overline{AD} = \dfrac{3-0}{0-5} = -\dfrac{3}{5}$

Opposite sides are parallel, but adjacent sides are not perpendicular. So, it is a parallelogram but not a rectangle.

24. radius = distance between (1, 2) and (5, 0)

$= \sqrt{(5-1)^2 + (0-2)^2} = \sqrt{20}$

So, the equation is $(x-1)^2 + (y-2)^2 = 20$.

25. $x^2 + 6x + y^2 - 4y = 12$

$x^2 + 6x + 9 + y^2 - 4y + 4 = 12 + 9 + 4$

$(x+3)^2 + (y-2)^2 = 25$

The circle has center (−3, 2) and radius 5.

26. $M = \left(\dfrac{0+2b}{2}, \dfrac{0+2c}{2}\right) = (b, c)$

$N = \left(\dfrac{2b+2a}{2}, \dfrac{2c+0}{2}\right) = (a+b, c)$

27. $\overline{MN}$ and $\overline{OB}$ are both horizontal segments and thus parallel. The length of $\overline{OB}$ is $2a$, and the length of $\overline{MN}$ is $|(a+b)-b| = a$.

So, $\overline{MN}$ is parallel to $\overline{OB}$ and half the length of $\overline{OB}$.