## LESSON 167

**1.** Tangent and radius are perpendicular.
$(r + 8)^2 = r^2 + 12^2$
$r = 5$

**2.** Tangent segments to a circle from a point are ≅.
$AE = AH = 8$
$BE = 15 - AE = 7$
$BF = BE = 7$
$CF = CG = 10$
$DG = DH = 5$
→ perimeter
$= AB + BC + CD + AD$
$= 15 + 17 + 15 + 13$
$= 60$

**3.** 180°; A diameter divides a circle into two semicircles. A semicircle measures 180°.

**4.** A radius perpendicular to a chord bisects the chord and its arc. A semicircle measures 180°.
$x = 6$
$y = 180 - 64 = 116$

**5.** A radius perpendicular to a chord bisects the chord.
base of right △ = 14/2 = 7
$r^2 = 4^2 + 7^2$
$r = \sqrt{65}$

**6.** Chords equidistant from the center are congruent.
$x = 9$

Congruent chords have congruent arcs. A circle measures 360°.

$y + y + 166 = 360$; $y = 97$

**7.** A circle measures 360°.
$a = 360 - 90 - 134 = 136$

An inscribed angle measures half its intercepted arc.
$b = a/2 = 136/2 = 68$

**8.** An angle inscribed in a semicircle is a right angle.
$m\angle B = 90°$

Base angles of an isosceles triangle are congruent.
$m\angle A = m\angle C = 45°$

A diameter is twice a radius.
$AC = 2(5\sqrt{2}) = 10\sqrt{2}$

A 45-45-90 triangle has sides in the ratio $1:1:\sqrt{2}$.
$AB = BC = 10$

**9.** An inscribed angle measures half its intercepted arc.
$m\angle 1 = (100 + 118)/2 = 109°$

Opposite angles of an inscribed quadrilateral are supplementary.

$m\angle 2 = 180 - 90 = 90°$
$m\angle 3 = 180 - m\angle 1 = 180 - 109 = 71°$

**10.** A chord-tangent angle measures half its intercepted arc.
$m\angle 1 = 154/2 = 77°$
$m\angle 2 = 180 - m\angle 1 = 180 - 77 = 103°$

**11.** A chord-chord angle measures half the sum of the intercepted arcs.
$m\angle 1 = (54 + 70)/2 = 62°$
$m\angle 2 = 180 - m\angle 1 = 180 - 62 = 118°$

**12.** A secant-secant angle measures half the difference of the intercepted arcs.
$m\angle 1 = (129 - 63)/2 = 33°$

**13.** A tangent-tangent angle measures half the difference of the intercepted arcs.
$m\angle 1 = ((360 - 117) - 117)/2 = 63°$

**14.** The product of the segments of one chord equals the product of the segments of the other chord.
$9x = 6(12)$; $x = 8$

**15.** The product of the secant segment and its external part is equal to the square of the tangent segment.
$x(x + 8) = 9^2$
$x^2 + 8x - 81 = 0$
$x = -4 + \sqrt{97}$

**16.** $\overline{OA} \cong \overline{OB} \cong \overline{OC} \cong \overline{OD}$ as radii of the same circle.
$\angle AOB \cong \angle COD$ as vertical angles.
So, $\triangle AOB \cong \triangle COD$ by SAS.

**17.** B, D

**18.** $m\widehat{BC} = 180 - m\widehat{CD} = 180 - 60 = 120°$
$m\widehat{AE} = 180 - m\widehat{AB} - m\widehat{DE} = 180 - 46 - 46 = 88°$

$m\angle 1 = 90°$ (inscribed in a semicircle)
$m\angle 2 = (m\widehat{AB} + m\widehat{DE})/2 = 46°$ (chord-chord angle)
$m\angle 3 = 90°$ (inscribed in a semicircle)
$m\angle 4 = m\widehat{CD}/2 = 30°$ (inscribed angle)
$m\angle 5 = m\widehat{BC}/2 = 60°$ (inscribed angle)
$m\angle 6 = m\widehat{CD}/2 = 30°$ (chord-tangent angle)
$m\angle 7 = (m\widehat{BAD} - m\widehat{CD})/2 = 60°$ (secant-tangent angle)

There are many ways to find these angle measures that are all correct. For example, you could use the Triangle Exterior Angle Theorem [32.2] to find $m\angle 7 = m\angle 3 - m\angle 6 = 90 - 30 = 60°$.