LESSON 172

1. P(multiple of 3) = P(3, 6, or 9) = 3/10
2. P(hit) = 15/25 = 3/5
3. The complement of "at least one heads" is "no heads."
   8 possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT
   P(no heads) = P(all tails) = P(TTT) = 1/8
   P(at least one heads) = 1 – P(no heads) = 1 – 1/8 = 7/8
4. P(correct) × P(correct) × P(correct) × P(correct)
   $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$
5. P(clear) × P(clear)
   $= \frac{6}{10} \times \frac{6}{10} = \frac{9}{25}$
6. 6 x 6 = 36 possible outcomes
   6 favorable outcomes: 11, 22, 33, …, 66
   P(same numbers) = 6/36 = 1/6
7. P(clear) × P(clear|clear)
   $= \frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$
8. P(club) × P(club|club)
   $= \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$
9. P(heart) + P(face) − P(heart and face)
   $= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{11}{26}$
10. swimming only = 22 – 7 = 15
    rock climbing only = 19 – 7 = 12
    swimming only or rock climbing only = 15 + 12 = 27
    P(swimming only or rock climbing only) = 27/40
11. P(vanilla|mint) = (vanilla and mint)/mint
    = 10/(10 + 15) = 2/5
12. P(coffee) = 26/40 = 13/20
13. P(tea|male) = (tea and male)/male = 6/18 = 1/3
14. P(red) = red/(green + yellow + red)
    = 80/(50 + 10 + 80) = 4/7
15. entire area = circle with radius 9 = $\pi(9)^2 = 81\pi$
    favorable area
    = circle with radius 6 – circle with radius 3
    = $36\pi - 9\pi = 27\pi$
    $P(\text{shaded region}) = \frac{\text{favorable area}}{\text{entire area}} = \frac{27\pi}{81\pi} = \frac{1}{3}$
16. 4 possible outcomes: HH, HT, TH, TT
    possible values of X: 0, 1, 2
    P(0) = P(no heads) = P(TT) = 1/4
    P(1) = P(one heads) = P(HT or TH) = 1/2
    P(2) = P(two heads) = P(HH) = 1/4
    $E(X) = 0 \times P(0) + 1 \times P(1) + 2 \times P(2)$
    = 0(1/4) + 1(1/2) + 2(1/4) = 1
    So, the expected value is 1.
17. Find the number of permutations of 5.
    $P(5, 5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5 \times 4 \times 3 \times 2 \times 1 = 120$
18. Find the number of permutations of 4 out of 6.
    $P(6, 4) = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360$
19. Find the number of combinations of 3 out of 12.
    $C(12, 3) = \frac{12!}{(12-3)!\,3!} = \frac{12!}{9!\,3!} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$
20. Two points determine a line. Because order does not matter, the number of lines is the number of combinations of 2 out of 10.
    $C(10, 2) = \frac{10!}{(10-2)!\,2!} = \frac{10!}{8!\,2!} = \frac{10 \times 9}{2 \times 1} = 45$
21. possible outcomes = P(5, 5)
    favorable outcomes = permutations of 4 remaining letters after placing 5 first = P(4, 4)
    probability $= \frac{P(4, 4)}{P(5, 5)} = \frac{4!}{5!} = \frac{1}{5}$
22. possible outcomes = C(10, 2)
    favorable outcomes = combinations of choosing 2 teens out of 4 = C(4, 2)
    probability $= \frac{C(4, 2)}{C(10, 2)} = \frac{6}{45} = \frac{2}{15}$
23. Emma and Brian must sit next to each other, so group them together and treat them as one person.
    possible outcomes = P(8, 8)
    favorable outcomes = permutations of 7 people with Emma-Brian + permutations of 7 people with Brian-Emma = P(7, 7) + P(7, 7) = P(7, 7) × 2
    probability $= \frac{P(7, 7) \times 2}{P(8, 8)} = \frac{7! \times 2}{8!} = \frac{1}{4}$

24. The vowels should be placed second and fourth.

possible outcomes = $P(5, 5)$

favorable outcomes = permutations of 3 consonants with A second and E fourth + permutations of 3 consonants with E second and A fourth = $P(3, 3) \times 2$

probability = $\frac{P(3, 3) \times 2}{P(5, 5)} = \frac{3! \times 2}{5!} = \frac{1}{10}$

You could use the counting principle.

possible outcomes = 5 × 4 × 3 × 2 × 1 = 120

favorable outcomes = 3 × 2 × 2 × 1 × 1 = 12

1st place: 3 consonants

2nd place: 2 vowels

3rd place: 3 – 1 = 2 consonants

4th place: 2 – 1 = 1 vowel

5th place: 3 – 2 = 1 consonant

probability = 12/120 = 1/10