LESSON 174

1. Tangent segments to a circle from a point are ≅.

$AD = AF = 10$ → perimeter

$BE = BD = 15$ $= AB + BC + AC$

$CE = CF = 9$ $= 25 + 24 + 19 = 68$

2. Congruent chords are equidistant from the center.

$x = 6/2 = 3$

A radius perpendicular to a chord bisects the chord.

base of right △ = 8/2 = 4

Use the Pythagorean Theorem or the 3-4-5 Pythagorean triple to find $y = 5$.

3. An inscribed angle measures half its intercepted arc.

$m\angle 1 = 118/2 = 59°$

A chord-chord angle measures half the sum of the intercepted arcs.

$m\angle 2 = (86 + 118)/2 = 102°$

4. An angle inscribed in a semicircle is a right angle.

$m\angle 1 = 90°$

An inscribed angle measures half its intercepted arc.

$m\angle 2 = 60/2 = 30°$

Angles in a triangle add up to 180°.

$m\angle 3 = 180 - m\angle 1 - m\angle 2 = 60°$

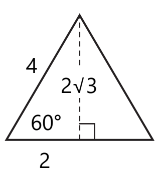
5. The product of the secant segment and its external part is equal to the square of the tangent segment.

$x^2 = 15(15 + 10 + 7)$; $x = 4\sqrt{30}$

The product of the segments of one chord equals the product of the segments of the other chord.

$14y = 7(10)$; $y = 5$

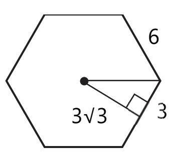
6.



An equilateral triangle can be divided into two congruent 30-60-90 triangles. Use a 30-60-90 triangle to find that the height is $2\sqrt{3}$.

area $= \frac{1}{2}bh = \frac{1}{2}(4)(2\sqrt{3}) = 4\sqrt{3}$

7.



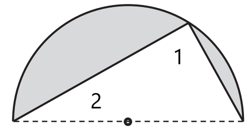
A regular hexagon is made up of 6 equilateral triangles. The apothem is the height of each equilateral triangle. Use a 30-60-90 triangle to find that the apothem is $3\sqrt{3}$.

area $= \frac{1}{2}sa \cdot n = \frac{1}{2}(6)(3\sqrt{3})(6) = 54\sqrt{3}$

8. arc length $= \frac{\theta}{360} \cdot 2\pi r = \frac{150}{360} \cdot 2\pi(12) = 10\pi$

area $= \frac{\theta}{360} \cdot \pi r^2 = \frac{150}{360} \cdot \pi(12)^2 = 60\pi$

9.



An angle inscribed in a semicircle is a right angle, so $m\angle 1 = 90°$. An inscribed angle measures half its intercepted arc, so $m\angle 2 = 60/2 = 30°$. So, the triangle is a 30-60-90 triangle with hypotenuse 20 and legs 10 and $10\sqrt{3}$.

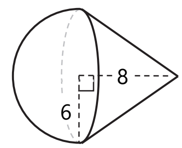
area = semicircle – triangle

$= \frac{1}{2}\pi(10)^2 - \frac{1}{2}(10)(10\sqrt{3}) = 50\pi - 50\sqrt{3}$

10. $SA = 2\pi r^2 + 2\pi rh = 2\pi(5)^2 + 2\pi(5)(6) = 110\pi$

$V = \pi r^2 h = \pi(5)^2(6) = 150\pi$

11.



The solid is a cone with a hemisphere on top. The hemisphere has radius 6. The cone has radius 6 and height 8.

12. Use the Pythagorean Theorem or a multiple of the 3-4-5 Pythagorean triple to find that the slant height of the cone is 10.

SA = hemisphere LA + cone LA

$= \frac{1}{2} \cdot 4\pi r^2 + \pi rl = \frac{1}{2} \cdot 4\pi(6)^2 + \pi(6)(10) = 132\pi$

V = hemisphere V + cone V

$= \frac{1}{2} \cdot \frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{1}{2} \cdot \frac{4}{3}\pi(6)^3 + \frac{1}{3}\pi(6)^2(8) = 144\pi + 96\pi = 240\pi$

13. area ratio $= 48/27 = 16/9 = 4^2/3^2$

height ratio = side ratio = 4/3

volume ratio $= 4^3/3^3 = 64/27$

14. tank $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12)^3 = 2304\pi$

$\approx 2304(22/7) \approx 7241$

time = volume / rate ≈ 7241/50 ≈ 145

So, it will take about 145 minutes.

15. cylinder $V = \pi r^2 h = \pi(2)^2(2) = 8\pi$

$\approx 8(22/7) \approx 25.1$

density = mass/volume ≈ 227/25.1 ≈ 9

So, the density is about 9 pounds per cubic foot.

16. population density = people/land area

= 504,000/60 = 8400

So, the population density is 8,400 people/mile^2.

17. center = midpoint between (−5, 2) and (3, −2)

$$= \left(\frac{-5+3}{2}, \frac{2-2}{2}\right) = (-1, 0)$$

radius = distance between (−1, 0) and (3, −2)

$$= \sqrt{(3-(-1))^2 + (-2-0)^2} = \sqrt{20}$$

So, the equation is $(x + 1)^2 + y^2 = 20$.

18. You need to move right and up to find P.

x-length of $\overline{AB}$ = 8 – (–7) = 15

x of P = x of A + (3/5)(x-length) = –7 + (3/5)15 = 2

y-length of $\overline{AB}$ = 4 – (–1) = 5

y of P = y of A + (3/5)(y-length) = –1 + (3/5)5 = 2

So, P is at (2, 2).

19. Find a line perpendicular to $\overline{AB}$ and passing through the midpoint of $\overline{AB}$.

slope of $\overline{AB}$ = 1/3

midpoint of $\overline{AB}$ = (2, 2)

perpendicular slope = –3

point-slope form: $y - 2 = -3(x - 2)$

slope-intercept form: $y = -3x + 8$

20. You can tell from the graph that it is not a rhombus. It looks like a parallelogram or a rectangle. Check the slopes of the sides. By the slope formula,

slope of $\overline{AB}$ = 3/2 slope of $\overline{BC}$ = −3/5

slope of $\overline{CD}$ = 3/2 slope of $\overline{AD}$ = −3/5

Opposite sides are parallel, but adjacent sides are not perpendicular. So, it is a parallelogram but not a rectangle.

21 ~ 22. Use your ruler and protractor to check the accuracy of your construction.

23. Use the complement rule.

P(sum is 5) = P(14, 23, 32, or 41) = 4/36 = 1/9

P(sum is not 5) = 1 – P(sum is 5) = 1 – 1/9 = 8/9

24. Let x = number of students who speak both.

Spanish only + French only + both + neither = 30

$(17 - x) + (14 - x) + x + 4 = 30$; $x = 5$

P(both) = 5/30 = 1/6

25. entire area = rectangle = 10(10) = 100

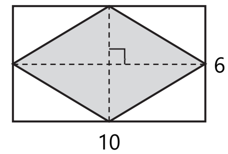
favorable area = circle = $\pi(5)^2 = 25\pi$

$$P(\text{shaded region}) = \frac{\text{favorable area}}{\text{entire area}} = \frac{25\pi}{100} = \frac{\pi}{4}$$

26. Order does not matter, so find the number of combinations of 3 out of 8.

$$C(8, 3) = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

27.



The area of the rhombus is half the area of the rectangle, so the probability is 1/2.