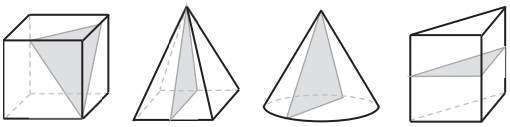
LESSON 179

1. A (vertical angles)
D (corresponding angles on parallel lines)

2. A, B, C, E
D is false because a straight angle does not have a supplement. F is false because a square is a rhombus but not all rhombuses are squares.

3. $m\angle ACB$ = 180 – 132 = 48° (supplementary angles)
$m\angle ABC = m\angle ACB$ = 48° (base angles of isosceles △)
$m\angle 1 = m\angle ABC$ = 48° (alternate interior angles)

4. A, B, C, D



5. If the tangent is 1, then the legs must be congruent. A right triangle with congruent legs is a 45-45-90 triangle.

$$\cos y^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

6. 2. Vertical angles are congruent.
3. AAS

7. 4 possible outcomes: HH, HT, TH, TT
3 favorable outcomes: HH, HT, TH
P(at least one heads) = 3/4

You could use the complement rule.
P(at least one heads) = 1 - P(no heads)
= 1 – P(TT) = 1 – 1/4 = 3/4

8. C, B, A, D

9. Use a 30-60-90 triangle to find that the height is $6\sqrt{3}$.

area = $\frac{1}{2} h(b_1 + b_2) = \frac{1}{2}(6\sqrt{3})(11 + 6 + 11) = 84\sqrt{3}$

10. Tangent segments to a circle from a point are ≅.
perimeter = (8 + 9) + (9 + 12) + (8 + 12) = 58

11. Let b be the base of the right triangle. Then the Pythagorean Theorem gives $b = 3\sqrt{3}$.
A radius perpendicular to a chord bisects the chord, so the length of the chord is $2b = 6\sqrt{3}$.

12. A, B, C, D

13. A midsegment is parallel to the third side.
∠1 and 76° are congruent as corresponding angles. ∠1 and ∠2 are congruent as alternate interior angles.
So, $m\angle 1 = m\angle 2$ = 76°.

14. Find a line perpendicular to $\overline{AB}$ and passing through the midpoint of $\overline{AB}$.
slope of $\overline{AB}$ = 1/4
midpoint of $\overline{AB}$ = (1, 2)
perpendicular slope = –4
point-slope form: $y - 2 = -4(x - 1)$
slope-intercept form: $y = -4x + 6$

15. A

16. $\angle ACD$ (angle formed by a tangent and a diameter)
$\angle ABC$ (angle inscribed in a semicircle)
$\angle CBD$ (angle supplementary to a right angle)

17. You need to move right and down to find P.
x-length of $\overline{AB}$ = 5 – 1 = 4
x of P = x of A + (1/4)(x-length) = 1 + (1/4)4 = 2
y-length of $\overline{AB}$ = 9 – 1 = 8
y of P = y of A − (1/4)(y-length) = 9 – (1/4)8 = 7
So, P is at (2, 7).

18. An intercepted arc measures twice its inscribed angle.
$m\widehat{AC} = 2m\angle B$ = 90°
An arc measure equals the measure of its central angle.
$m\angle APC = m\widehat{AC}$ = 90°
Sector APC has radius 4 and angle 90°.
area = $\frac{\theta}{360} \cdot \pi r^2 = \frac{90}{360} \cdot \pi(4)^2 = 4\pi$

19. D; A composition of reflections over two intersecting lines is a rotation.

20. A, C
B is false because a circumcenter is equidistant from the vertices of its triangle. D is false because an incenter is where the angle bisectors of a triangle meet.

21. $ACDE$ is a rectangle, so $AE = CD$ = 5 and $ED = AC$ = 16.

$$\tan 18^\circ = \frac{BC}{AC} = \frac{BC}{16}$$

BC = 16 tan 18° ≈ 5.2
$BD = BC + CD$ ≈ 5.2 + 5 = 10.2
So, the tree is about 10.2 ft tall.

22. Let x = number of students who take both.
Biology only + Chemistry only + both + neither = 40
$(25 - x) + (20 - x) + x + 7 = 40$; $x = 12$
P(Chemistry only) = (20 – 12)/40 = 1/5 = 20%

23. Tangent and radius are perpendicular. Angles in a quadrilateral add up to 360°.
$90 + 90 + x + y = 360$; $x + y = 180$
So, the value of $x + y$ is always 180.

24. center: (−3, 1), radius = 2

So, the equation is $(x + 3)^2 + (y - 1)^2 = 4$.

25. Use the Altitude Rule [68.1].

$9^2 = x(3x)$; $x = \sqrt{27} = 3\sqrt{3}$

So, the shorter segment is $3\sqrt{3}$ cm.

26. $\triangle EGH$ is a right triangle. The Pythagorean Theorem or the 5-12-13 Pythagorean triple gives $EG = 13$.

Because the diagonals of a rectangle are congruent, $FH = EG = 13$.

27. The product of the secant segment and its external part is equal to the square of the tangent segment.

$x^2 = 5(5 + 8)$; $x = \sqrt{65}$

28. B

A can use HL. C can use ASA. D can use SSS.

29. Any three points on a circle cannot be collinear. Three non-collinear points determine a triangle. Because order does not matter, the number of triangles is the number of combinations of 3 out of 8.

$$C(8, 3) = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

30. Use the Pythagorean Theorem or the distance formula to find the length of each side.

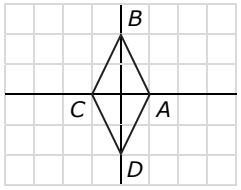
$A(0, 2)$, $B(4, -1)$, $C(-3, -2)$

$AB = \sqrt{(4-0)^2 + (-1-2)^2} = 5$

$BC = \sqrt{(-3-4)^2 + (-2-(-1))^2} = 5\sqrt{2}$

$AC = \sqrt{(-3-0)^2 + (-2-2)^2} = 5$

So, the perimeter is $10 + 5\sqrt{2}$ units.

31. 

All sides are congruent, so it is a rhombus. The answer is B.

32. Use the Triangle Side Splitter Theorem [69.1].

$$\frac{20}{4x} = \frac{5x}{9} \quad \rightarrow \quad 4x(5x) = 20(9) \quad \rightarrow \quad x = 3$$

33. aquarium $V = Bh = 4(3)(2) = 24$

density = mass/volume

$62 = x/24$; $x = 1488$

So, the aquarium can hold 1,488 pounds of water.

34. balloon $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi$

$\approx 288(22/7) \approx 905$ in^3

time = volume / rate ≈ 905/20 = 45

So, it will take about 45 minutes.

35. entire area = circle with radius 6 = $\pi(6)^2 = 36\pi$

favorable area = a rhombus with diagonals 12 and 12

$$= \frac{1}{2}(12)(12) = 72$$

$$P(\text{shaded region}) = \frac{\text{favorable area}}{\text{entire area}} = \frac{72}{36\pi} = \frac{2}{\pi}$$