LESSON 180

1. A (alternate interior angles)
B (corresponding angles)

2. C

3. B

4. A, B, D

Counterexamples may vary. Sample(s):

A) The statement is false when $a = 1$, $b = 2$, $c = 0$.

B) A 45-45-90 triangle and a 60-60-60 triangle are not similar because their angles are not congruent.

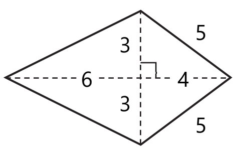
D) Side lengths 1, 1, and 5 cannot form a triangle.

5. $m\angle AEB = 180 - 90 - 52 = 38°$ ($\triangle$ angle sum = 180)

$m\angle ADC = m\angle AEB = 38°$ (corresponding angles)

$m\angle 1 = 180 - m\angle ADC = 142°$ (supplementary angles)

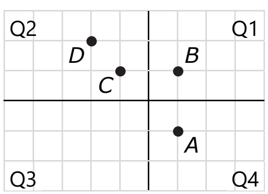
6.



The diagonals of a kite are perpendicular. Use the Pythagorean Theorem or the 3-4-5 Pythagorean triple to find that the length of the vertical diagonal is 3 + 3 = 6.

area = $\frac{1}{2} d_1 d_2 = \frac{1}{2}(4 + 6)(3 + 3) = 30$ units^2

7.



Use any point in Q4. The reflection maps A to B, the rotation maps B to C, and the dilation maps C to D.

So, the resulting image is in Quadrant 2.

8. D; An orthocenter is the point where the altitudes of a triangle intersect.

9. 3. If lines are parallel, then alternate interior angles are congruent.

4. Reflexive Property

5. ASA

10. Use the Three Parallel Lines Theorem [69.2].

$\frac{8}{6x} = \frac{5x}{15}$ → $(6x)(5x) = 8(15)$ → $x = 2$

11. C

12. B, D, F

13. A radius perpendicular to a chord bisects the chord, so the base of the right triangle is 20/2 = 10.

Let r be the radius of the circle. The right triangle has base 10, height 5, and hypotenuse r. By the Pythagorean Theorem,

$r^2 = 5^2 + 10^2$

$r = \sqrt{125} = 5\sqrt{5}$

So, the radius of the circle is $5\sqrt{5}$ inches.

14. D

15. An intercepted arc measures twice its inscribed angle.

$m\widehat{AC} = 2m\angle B = 50°$

An arc measure equals the measure of its central angle.

$m\angle APC = m\widehat{AC} = 50°$

Sector APC has radius 18 and angle 50°.

area = $\frac{\theta}{360} \cdot \pi r^2 = \frac{50}{360} \cdot \pi(18)^2 = 45\pi$

16. $x°$ and $y°$ are complementary. The cosine of an acute angle is equal to the sine of its complement.

$\cos y° = \sin x° = 5/13$

17. The circle has center (1, 4) and radius $\sqrt{5}$. By the distance formula,

distance between (1, 4) and (−1, 2)

$= \sqrt{(-1-1)^2 + (2-4)^2} = \sqrt{8}$

The distance is greater than the radius ($\sqrt{8} > \sqrt{5}$), so the point is outside the circle.

18. Parallel lines have the same slope.

original slope = 4

parallel slope = 4

point-slope form: $y - 1 = 4(x + 1)$

slope-intercept form: $y = 4x + 5$

19. Use the Secant-Secant Product Theorem [99.2].

$8(8 + x) = 7(7 + x + 3)$

$64 + 8x = 7x + 70$

$x = 6$

20. The diagonals of a parallelogram bisect each other.

$4x = 8$ → $10 - x = 5y - 2$

$x = 2$ → $10 - 2 = 5y - 2$

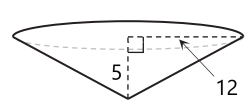
$y = 2$

21. P is the centroid and divides $\overline{BD}$ in the ratio 2:1.

$PD = BP/2 = 8/2 = 4$

$BD = BP + PD = 8 + 4 = 12$

22.



The solid is an upside-down cone with radius 12 and height 5.

volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(12)^2(5) = 240\pi$

23. Use the Pythagorean inequality theorems.
A is a right triangle because $5^2 = 3^2 + 4^2$.
B is an acute triangle because $6^2 < 4^2 + 5^2$.
C is a right triangle because $13^2 = 5^2 + 12^2$.
D is an obtuse triangle because $15^2 > 8^2 + 2^2$.
So, the answer is B.

24. C, E, F

25. You need to move right and up to find P.
x-length of $\overline{AB}$ = 8 – (–2) = 10
x of P = x of A + (3/5)(x-length) = –2 + (3/5)10 = 4
y-length of $\overline{AB}$ = 5 – 0 = 5
y of P = y of A + (3/5)(y-length) = 0 + (3/5)5 = 3
So, P is at (4, 3).

26. The preimage has center (2, –7) and radius 5.
The image has center (2, 7) and radius 5.
So, the equation is $(x - 2)^2 + (y - 7)^2 = 25$.

27. Use the complement rule.
P(sum is 3) = P(12 or 21) = 2/36 = 1/18
P(sum is not 3) = 1 – P(sum is 3) = 1 – 1/18 = 17/18

28. 10 × 9 × 8 = 720 ways

29. By the slope formula,

slope of $\overline{AB}$ = –2	slope of $\overline{BC}$ = 1/2
slope of $\overline{CD}$ = –2	slope of $\overline{AD}$ = 1/2

By the Pythagorean Theorem or the distance formula,

length of $\overline{AB} = 2\sqrt{5}$	length of $\overline{AB} = 2\sqrt{5}$
length of $\overline{CD} = 2\sqrt{5}$	length of $\overline{CD} = 2\sqrt{5}$

Adjacent sides are perpendicular (the product of the slopes of adjacent sides is –1), so it is a rectangle. All sides are congruent, so it is a rhombus. A quadrilateral that is a rectangle and a rhombus is a square.
The answer is A.

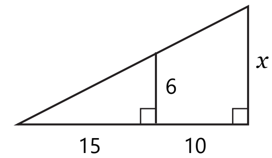
30. tank surface area = $4\pi r^2 = 4\pi(15)^2 = 900\pi$
≈ 900(22/7) ≈ 2829 ft^2
number of cans ≈ 2829/180 ≈ 16
So, about 16 cans will be needed.

31. population density = people/land area
979 = x/11787
x = 11,539,473 ≈ 11,539,000
So, the population is about 11,539,000 people.

32. Let x be the number of students who play neither.
soccer only + baseball only + both + neither = 50
(30 – 14) + (22 – 14) + 14 + x = 50
x = 12
So, P(neither) = 12/50 = 6/25.

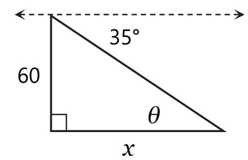
33 ~ 34. *Diagrams are not drawn to scale.*

33. $\frac{15}{15+10} = \frac{6}{x}$



15x = 6(15 + 10)
x = 10
The street lamp is 10 feet tall.

34. θ = 35°



tan 35° = 60/ x
x = 60 / tan 35° ≈ 85.7
The boat is about 85.7 ft away from the lighthouse.

35. entire area = circle with radius 12 = $\pi(12)^2 = 144\pi$
favorable area
= circle with radius 6 – circle with radius 3
= $36\pi - 9\pi = 27\pi$

$$P(\text{shaded region}) = \frac{\text{favorable area}}{\text{entire area}} = \frac{27\pi}{144\pi} = \frac{3}{16}$$