LESSON 27 Indirect Proofs

Given: $3x - 2 > 16$
Prove: $x > 6$

Assume $x > 6$ is false. Then $x \leq 6$.

Use algebra to try to make $x \leq 6$ equal to the given statement. Multiplying each side by 3 and subtracting 2 would produce $3x - 2 \leq 16$. This contradicts the given statement that $3x - 2 > 16$.

Therefore, our assumption is wrong and $x > 2$ must be true.

Given: $7x - 4$ is odd.
Prove: x is odd.

Assume x is even. 2 times any number produces an even number, so we can write our assumption as $n = 2k$ for some integer k.

If $n = 2k$, then $7x - 4 = 7(2k) - 4 = 14k - 4 = 2(7k - 2)$, which is even. This contradicts the given statement.

Therefore, our assumption is wrong and x must be odd.

Given: $\triangle ABC$ is equiangular.
Prove: $m\angle A = 60°$

Assume $m\angle A \neq 60°$.

An equiangular triangle has three congruent angles, so the interior angle sum of $\triangle ABC = 3(m\angle A)$. If $m\angle A \neq 60°$, then $3(m\angle A)$ cannot be 180°. This contradicts the fact that the interior angle sum of a triangle is 180°.

Therefore, our assumption that $m\angle A \neq 60°$ cannot be true, which proves that $m\angle A = 60°$.

Given: $ABCDEF$ is a regular hexagon.
Prove: $\angle A$ is obtuse.

Assume $\angle A$ is not obtuse. Then $m\angle A \leq 90°$.

A regular hexagon has six congruent angles, so the interior angle sum of $ABCDEF = 6(m\angle A)$. If $m\angle A \leq 90$, then $6(m\angle A) \leq 540°$. This contradicts the fact that the interior angle sum of a hexagon is $180(6 - 2) = 720°$.

Therefore, our assumption is wrong and $\angle A$ must be obtuse.