LESSON 85

1. A midsegment is half the length of the third side.

$3x - 8 = 2(14)$; $x = 12$

2. A midsegment is parallel to the third side. Corresponding angles on parallel lines congruent.

$5x + 1 = 56$; $x = 11$

3. Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

$9x - 8 = 4x + 7$; $x = 3$

4. An angle bisector divides an angle into two congruent angles. Angles in a triangle add up to 180°.

$90 + 56 + 2(x - 9) = 180$; $x = 26$

5. A median divides the side to which it is drawn into two congruent segments.

$7x - 8 = 3x$; $x = 2$

6. An altitude forms right angles with the side to which it is drawn. Angles in a triangle add up to 180°.

$90 + 56 + (x + 18) = 180$; $x = 16$

7. perpendicular bisector

8. A circumcenter is equidistant from the vertices of its triangle.

$YC = ZC = XC = 10$

9. median

10. A centroid divides a median in the ratio 2:1.

$PE = PB/2 = 12/2 = 6$

$BE = 3PE = 3(6) = 18$

11. no; 4 + 6 > 10 is false. The sum of two sides of a triangle must be greater than the third side.

12. The sum of two sides of a triangle must be greater than the third side. Let x be the third side.

$x + 3 > 5$	$x + 5 > 3$	$3 + 5 > x$
$x > 2$	$x > -2$	$x < 8$

Combine the inequalities to get $2 < x < 8$. So, the third side must be longer than 2 and shorter than 8.

13. The longer side has the larger opposite angle.

$\angle C$ is bigger than $\angle A$ because $AB > BC$.

14. Statements (Reasons)

1. midsegments $\overline{PQ}$, $\overline{QR}$, $\overline{PR}$ (Given)
2. $PQ = \frac{1}{2}AB$, $QR = \frac{1}{2}BC$, $PR = \frac{1}{2}AC$
   (A midsegment is half the length of the third side. See Theorem 46.1.)
3. perimeter of $\triangle PQR = PQ + QR + PR$, perimeter of $\triangle ABC = AB + BC + AC$
   (Def. of perimeter)
4. perimeter of $\triangle PQR = \frac{1}{2}AB + \frac{1}{2}BC + \frac{1}{2}AC$
   (Substitution Property)
5. perimeter of $\triangle PQR = \frac{1}{2}(AB + BC + AC)$
   (Distributive Property)
6. perimeter of $\triangle PQR = \frac{1}{2}$ perimeter of $\triangle ABC$
   (Substitution Property)

15. parallelogram, rhombus, rectangle, square

16. rectangle, square

17. rhombus, square, kite

18. Opposite angles are congruent.

$m\angle A = m\angle C = 70°$

Consecutive angles are supplementary.

$m\angle B = m\angle D = 180 - 70 = 110°$

19. Opposite sides are congruent.

$AB = CD$; $3x = 2x + 18$; $x = 18$

Diagonals bisect each other.

$AC = 2AE$; $7y + 30 = 2(5y)$; $y = 10$

20. $\triangle EPF \cong \triangle EPH \cong \triangle GPF \cong \triangle GPH$

All sides are congruent. Diagonals bisect each other and are perpendicular. So, the four right triangles are congruent by SSS, SAS, or HL.

21. Diagonals bisect opposite angles.

$m\angle F = 2(m\angle EFP) = 2(56) = 112°$

Opposite angles are congruent.

$m\angle H = m\angle F = 112°$

Consecutive angles are supplementary.

$m\angle E = m\angle G = 180 - 112 = 68°$

22. $\triangle PQR \cong \triangle QPS \cong \triangle SRQ \cong \triangle RSP$

All angles are right angles. Opposite sides are congruent. Diagonals are congruent. So, the four right triangles are congruent by SSS, SAS, or HL.

23. Diagonals are congruent.

$PR = QS$; $x + 5 = 3x - 7$; $x = 6$

$PR = 6 + 5 = 11$

24. Base angles are congruent.
$m\angle Z = m\angle Y = 68°$

Non-base angles are supplementary.
$m\angle W = m\angle X = 180 - 68 = 112°$

25. The midsegment is parallel to the bases.
Corresponding angles on parallel lines congruent.
$m\angle WMN = m\angle Z = 68°$
$m\angle XNM = m\angle Y = 68°$

26. The midsegment is half the sum of the bases.
$MN = (WX + ZY)/2 = (15 + 27)/2 = 21$

27. Angles in a triangle add up to 180°. See $\triangle STV$.
$m\angle S = 180 - m\angle STP - m\angle SVP = 180 - 45 - 30 = 105°$

Non-vertex angles are congruent.
$m\angle U = m\angle S = 105°$

28. Vertex angles are bisected by a diagonal.
$m\angle UTP = m\angle STP = 45°$
$m\angle UVP = m\angle SVP = 30°$

29. Statements (Reasons)

1. $\square ABCD$ (Given)
2. $\overline{BA} \parallel \overline{CD}, \overline{BC} \parallel \overline{AD}$ (Def. of parallelogram)
3. $\angle BAC \cong \angle DCA, \angle BCA \cong \angle DAC$ (Alternate interior ∠s on parallel lines are ≅.)
4. $\overline{AC} \cong \overline{CA}$ (Reflexive Property)
5. $\triangle ABC \cong \triangle CDA$ (ASA)
6. $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}$ (CPCTC)