This packet contains Honors Geometry material taken from EP Math Geometry Workbook. To take Honors Geometry online, use the online course and, additionally, complete the problems in this packet. The answers can be found at the end of this packet.

☐ LESSON 8

Solve.

1. True or false? Angles in a linear pair are complementary.

2. True or false? The supplement of an acute angle is obtuse.

3. True or false? Complementary angles are both acute angles.

4. True or false? Two obtuse angles can be supplementary to each other.

5. Two complementary angles are congruent. What is the measure of each angle?

6. Two congruent angles form a linear pair. What is the measure of each angle?

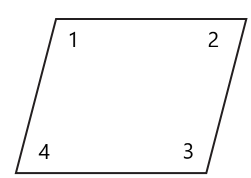
7. Two vertical angles are complementary. What is the measure of each angle?

8. An angle is the supplement of a right angle. What is the measure of the angle?

☐ LESSON 9

The diagram shows a parallelogram whose opposite sides are parallel.

1. Explain why any two adjacent angles of a parallelogram are supplementary.

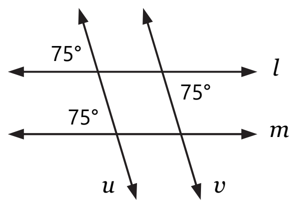


2. Explain why the sum of the interior angles of a parallelogram is always 360°. You will learn this angle sum property of a quadrilateral in a later lesson.

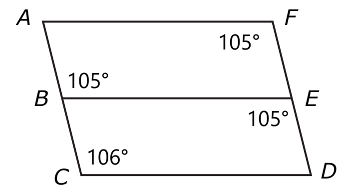
☐ LESSON 10

Identify all pairs of parallel lines or segments. Explain.

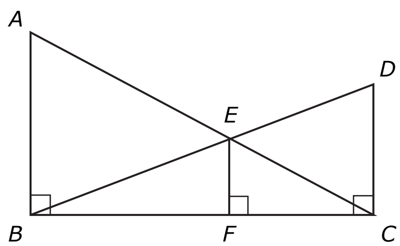
1.



2.



☐ LESSON 11



In the diagram, $m\angle A = 62°$ and $m\angle D = 70°$. Find the measure of each angle.

1. $m\angle ECF$

2. $m\angle EBF$

3. $m\angle ABE$

4. $m\angle AED$

☐ LESSON 13

Find the number of sides of each *regular* polygon.

1. Each interior angle measures 150°.

2. Each exterior angle measures 72°.

☐ **LESSON 16**

Find the point that gets mapped to the origin after each translation.

1. $T_{0,-2}$

2. $\langle -4, 0 \rangle$

3. $(x, y) \rightarrow (x - 2, y + 5)$

☐ **LESSON 17**

Solve.

1. A quadrilateral with vertices $P(5, 0)$, $Q(7, 1)$, $R(5, 3)$ and $S(3, 1)$ is translated 6 units left and reflected over the x-axis. How many vertices of the image are in Quadrant III?

2. Point $A(5, -8)$ is translated to a new position $(-2, -3)$. If you apply the same transformation to point $B(-3, 1)$, what would be the new coordinates of point B?

3. Point $P(2, 5)$ is translated and reflected over $y = x$ to a new position $(6, -1)$. If you apply the same transformations to point $Q(4, -3)$, what would be the new coordinates of point Q?

☐ LESSON 18

Solve.

1. Point $P(2, 5)$ is translated, reflected over $y = x$, and rotated 180° counterclockwise to a new position (2, 3). If you apply the same transformations to point $Q(9, 7)$, what would be the new coordinates of point Q?

2. A triangle with vertices $L(0, 0)$, $M(3, 0)$, and $N(0, 2)$ is rotated 180° counterclockwise about the origin, reflected over the y-axis, and translated 4 units left and 3 units up. In which quadrant is the image?

☐ LESSON 21

Solve.

1. What single transformation is the same as a dilation about the origin with a scale factor of -1?

Derive a single rule for each composite transformation. For example, a translation of 1 unit left and 1 unit up followed by a reflection over the line $y = x$ will map (x, y) to $(x - 1, y + 1)$ and then to $(y + 1, x - 1)$. So you can write its rule as $(x, y) \rightarrow (y + 1, x - 1)$.

2. Translate 3 units down, then reflect over the x-axis.

3. Reflect over the line $y = x$, then dilate about the origin by a scale factor of 2.

Do the problem(s) on the next page.

4. Identify each statement as true or false. If false, explain why.

A) A translation creates a mirror image of the original figure.

B) A reflection moves every point of the figure the same distance in the same direction.

C) A reflection over the x-axis changes the signs on the y-coordinates.

D) The image of a figure after a rotation is congruent to the original figure.

E) A 90° clockwise rotation is the same as a 90° counterclockwise rotation.

F) A dilation does not change angle measures.

G) A dilation with a scale factor of 3 makes a figure three times smaller.

H) A composition of translations, reflections, and rotations changes the size of a figure.

I) A 180° rotation about the origin is the same as two reflections over the x- and y-axes.

J) A composition of reflections over two intersecting lines can be described as a single reflection.

☐ LESSON 23

Study the examples and make a conjecture about each value.

1. the sum of three consecutive integers

$5 + 6 + 7 = 18 = 6 \times 3$

$7 + 8 + 9 = 24 = 8 \times 3$

$10 + 11 + 12 = 33 = 11 \times 3$

$123 + 124 + 125 = 337 = 124 \times 3$

2. the sum of the first n positive odd integers

$1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

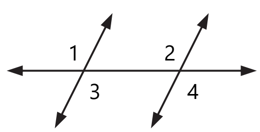
$1 + 3 + 5 + 7 = 16 = 4^2$

☐ LESSON 29

Write a proof using any format. Use the theorems you have just learned.

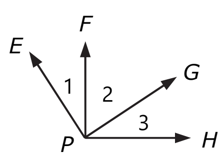
1. Given: $\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$



2. Given: $\overrightarrow{PE} \perp \overrightarrow{PG}$, $\overrightarrow{PF} \perp \overrightarrow{PH}$

Prove: $\angle 1 \cong \angle 3$



Do the problem(s) on the next page.

To fully prove Theorems 29.2 and 29.3, you also need to prove the cases of angles complementary to congruent angles and angles supplementary to congruent angles. Write a two-column proof to prove each case.

3. Given: $\angle 1$ is complementary to $\angle 3$.
$\angle 2$ is complementary to $\angle 4$.
$\angle 3 \cong \angle 4$

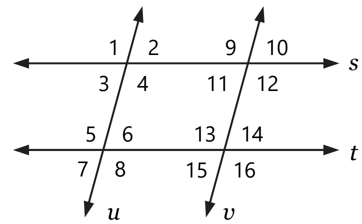
Prove: $\angle 1 \cong \angle 2$

4. Given: $\angle 1$ is supplementary to $\angle 3$.
$\angle 2$ is supplementary to $\angle 4$.
$\angle 3 \cong \angle 4$

Prove: $\angle 1 \cong \angle 2$

☐ LESSON 30

Use the diagram on the right to write a proof.



1. Given that $s \parallel t$ and $u \parallel v$, prove that $\angle 1 \cong \angle 13$.

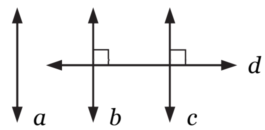
2. Given that $s \parallel t$ and $u \parallel v$, prove that $\angle 6 \cong \angle 11$.

3. Given that $s \parallel t$ and $\angle 3 \cong \angle 14$, prove that $u \parallel v$.

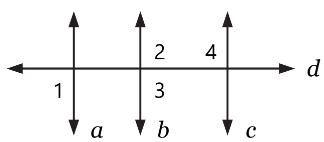
☐ LESSON 31

Write a proof using any format.

1. Given: $a \parallel b$, $b \perp d$, $c \perp d$
   Prove: $a \parallel c$



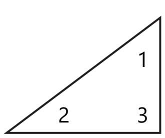
2. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
   Prove: $a \parallel c$



☐ LESSON 32

Complete the proof that the two acute angles in a right triangle are complementary.

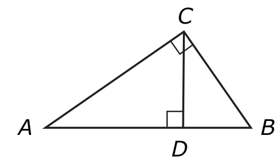
1. Given: $\angle 3$ is a right angle.
   Prove: $\angle 1$ and $\angle 2$ are complementary.



STATEMENTS	REASONS
1. $\angle 3$ is a right angle.	1. Given
2. $m\angle 3 = 90°$	2. Def. of right angle
3. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	3.
4. $m\angle 1 + m\angle 2 + 90 = 180°$	4.
5. $m\angle 1 + m\angle 2 = 90°$	5.
6. $\angle 1$ & $\angle 2$ are complementary.	6. Def. of complementary angles

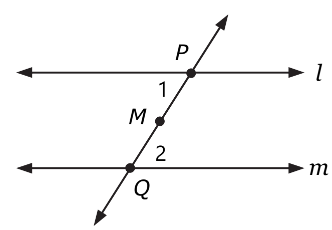
Do the problem(s) on the next page.

2. In the diagram, $\overline{AC} \perp \overline{BC}$ and $\overline{CD} \perp \overline{AB}$. Write a two-column proof to prove that $\angle B \cong \angle ACD$. (*Hint*: Use your previous proof and the Congruent Complements Theorem [29.2].)



☐ **LESSON 33**

Write a proof of the Alternate Interior Angles Theorem [30.2] using transformations.



1. Given: $l \parallel m$, M is the midpoint of $\overline{PQ}$.

Prove: $\angle 1 \cong \angle 2$

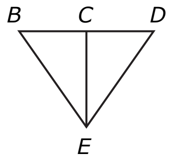
Hint: Use a 180° rotation about M.

☐ LESSON 36

Write a proof using any format.

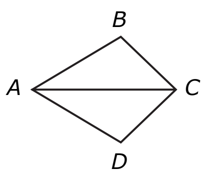
1. Given: $\overline{BD} \perp \overline{CE}$, C bisects $\overline{BD}$.

   Prove: $\triangle BCE \cong \triangle DCE$



2. Given: $ABCD$ is a kite.

   Prove: $\triangle ABC \cong \triangle ADC$



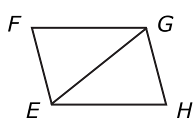
Hint: Use the definition of kite.

☐ LESSON 37

Write a proof using any format.

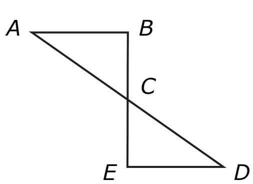
1. Given: $\overline{FG} \parallel \overline{EH}$, $\overline{FE} \parallel \overline{GH}$

   Prove: $\triangle EFG \cong \triangle GHE$



2. Given: $\overline{AB} \perp \overline{BE}$, $\overline{DE} \perp \overline{BE}$,
   $\overline{AB} \cong \overline{DE}$

   Prove: $\triangle ABC \cong \triangle DEC$

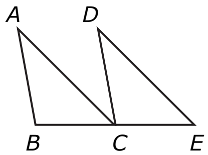


☐ LESSON 38

Write a proof using any format.

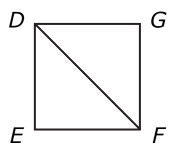
1. Given: $\overline{AB} \parallel \overline{DC}$, $\overline{AC} \parallel \overline{DE}$,
C bisects $\overline{BE}$.

Prove: $\overline{AC} \cong \overline{DE}$



2. Given: $DEFG$ is a square.

Prove: $\angle EDF \cong \angle GDF$



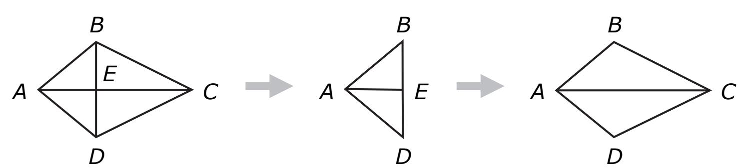
Hint: Use the definition of square.

☐ LESSON 39

Let's try more challenging problems, where there are two pairs of triangles to prove congruent. Complete the proof.

1. Given: $\overline{AC} \perp \overline{BD}$,
$\overline{AC}$ bisects $\overline{BD}$.

Prove: $\angle B \cong \angle D$



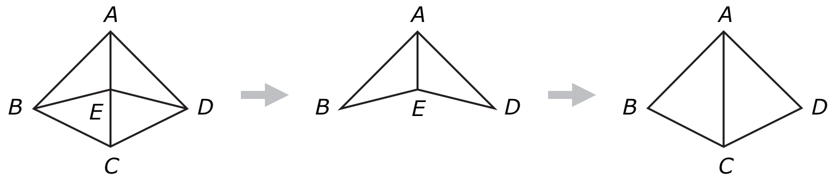
	STATEMENTS	REASONS
	1. $\overline{AC} \perp \overline{BD}$	1. Given
	2. $\angle AEB$, $\angle AED$ right ∠s.	2. Def. of ⊥ lines
A	3. $\angle AEB \cong \angle AED$	3.
	4. $\overline{AC}$ bisects $\overline{BD}$.	4. Given
S	5. $\overline{BE} \cong \overline{DE}$	5.
S	6. $\overline{AE} \cong \overline{AE}$	6.

	STATEMENTS (cont.)	REASONS (cont.)
	7. $\triangle AEB \cong \triangle AED$	7.
S	8. $\overline{AB} \cong \overline{AD}$	8. CPCTC
A	9. $\angle BAE \cong \angle DAE$	9. CPCTC
S	10. $\overline{AC} \cong \overline{AC}$	10.
	11. $\triangle ABC \cong \triangle ADC$	11.
	12. $\angle B \cong \angle D$	12.

Do the problem(s) on the next page.

Write a proof using any format.

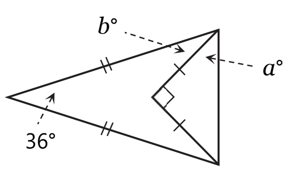
2. Given: $\overline{AB} \cong \overline{AD}$,
$\overline{BE} \cong \overline{DE}$
Prove: $\triangle ABC \cong \triangle ADC$



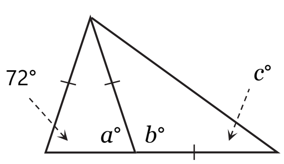
☐ LESSON 40

Find the values of the variables.

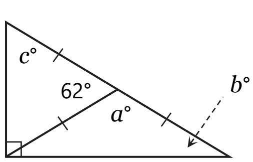
1.



2.



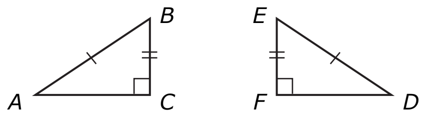
3.



☐ LESSON 42

The HL Congruence Theorem can also be proved using the Pythagorean Theorem. Read aloud the Pythagorean Theorem in Lesson 74, then complete the proof.

1. Given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$
$\angle C$ and $\angle F$ are right angles.
Prove: $\triangle ABC \cong \triangle DEF$



	STATEMENTS	REASONS
SS	1. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$	1. Given
	2. $AB = DE$, $BC = EF$	2.
	3. $AC^2 + BC^2 = AB^2$ $DF^2 + EF^2 = DE^2$	3. Pythagorean Theorem [74.1]
	4. $AC^2 = AB^2 - BC^2$ $DF^2 = DE^2 - EF^2$	4.

	STATEMENTS (cont.)	REASONS (cont.)
	5. $AC^2 = DE^2 - EF^2$	5. Substitution Prop.
	6. $AC^2 = DF^2$	6.
	7. $AC = DF$	7. Take square roots.
S	8. $\overline{AC} \cong \overline{DF}$	8.
	9. $\triangle ABC \cong \triangle DEF$	9.

☐ LESSON 44

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. $\triangle XYZ$ is transformed to produce $\triangle X'Y'Z'$. What transformation was applied to $\triangle XYZ$ if $\triangle XYZ \cong \triangle X'Y'Z'$, $\overline{XX'} \parallel \overline{YY'} \parallel \overline{ZZ'}$, and $XX' = YY' = ZZ'$?

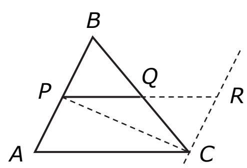
2. Line $y = x + 1$ is translated 3 units up and then reflected over the y-axis. Write an equation of the image in slope-intercept form. (*Hint*: Transform the x- and x- intercepts.)

☐ **LESSON 46**

Prove Theorem 46.1. Give a reason or reasons for each statement. Note that you may need to give more than one reason to justify a statement.

1. Given: $\overline{BP} \cong \overline{PA}, \overline{BQ} \cong \overline{QC}$

Prove: $\overline{PQ} \parallel \overline{AC}, PQ = \frac{1}{2}AC$



Draw a line through C parallel to $\overline{AB}$. Extend $\overline{PQ}$ to intersect at R. Draw $\overline{PC}$ as well. Then you have:

(1) $\triangle BQP \cong \triangle CQR$

(2) $\overline{QP} \cong \overline{QR}, \overline{PB} \cong \overline{RC}$

(3) $\overline{PA} \cong \overline{RC}$

(4) $\triangle APC \cong \triangle RCP$

(5) $\angle PCA \cong \angle CPR$

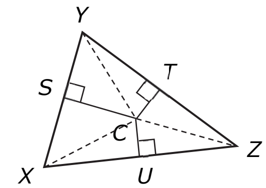
(6) $\overline{PQ} \parallel \overline{AC}$

(7) $\overline{PR} \cong \overline{AC}$

(8) $PQ = AC/2$ (or $AC = 2PQ$)

☐ **LESSON 47**

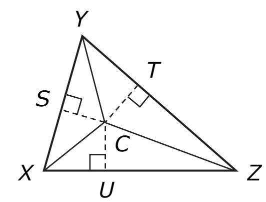
Prove Theorem 47.2. (*Hint*: Use Theorem 47.1.)



1. Given: $\overline{CS}$, $\overline{CT}$, and $\overline{CU}$ are the perpendicular bisectors of $\triangle XYZ$.
   Prove: $CX = CY = CZ$

☐ **LESSON 48**

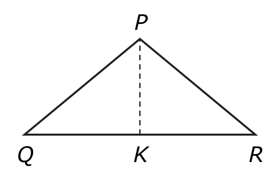
Prove Theorem 48.2. (*Hint*: Use Theorem 48.1.)



1. Given: $\overline{CX}$, $\overline{CY}$, and $\overline{CZ}$ are the angle bisectors of $\triangle XYZ$.
   $\overline{XY} \perp \overline{CS}$, $\overline{YZ} \perp \overline{CT}$, $\overline{XZ} \perp \overline{CU}$
   Prove: $CS = CT = CU$

☐ **LESSON 49**

$\triangle PQR$ is isosceles with $\overline{PQ} \cong \overline{PR}$. Explain why each statement is true.



1. If $\overline{PK}$ is a median, then it is also an angle bisector.

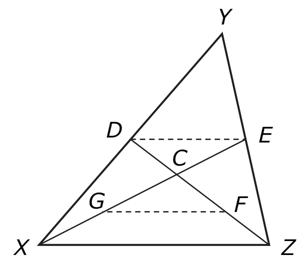
2. If $\overline{PK}$ is an angle bisector, then it is also a median.

Do the problem(s) on the next page.

Prove Theorem 49.1. Give a reason or reasons for each statement.

3. Given: $\overline{XE}$ and $\overline{ZD}$ are medians of $\triangle XYZ$.

Prove: $CX = 2CE, CZ = 2CD$



Draw $\overline{GF}$ so that G is the midpoint of $\overline{CX}$ and F is the midpoint of $\overline{CZ}$. Draw $\overline{DE}$ as well. Then you have:

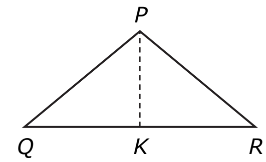
(1) $\overline{DE} \parallel \overline{GF} \parallel \overline{XZ}$ (2) $\overline{DE} \cong \overline{GF}$ (3) $\triangle CDE \cong \triangle CFG$

(4) $CE = CG = GX$ (5) $CX = 2CE$ (6) $CZ = 2CD$

This proves that any two medians of a triangle divide each other in the ratio 2:1.

☐ LESSON 50

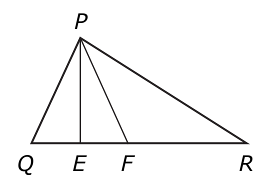
$\triangle PQR$ is isosceles with $\overline{PQ} \cong \overline{PR}$. Explain why each statement is true.



1. If $\overline{PK}$ is a median, then it is also an altitude.

2. If $\overline{PK}$ is an altitude, then it is also a median.

$\triangle PQF$ is isosceles with $\overline{PQ} \cong \overline{PF}$. $\triangle FPR$ is isosceles with $\overline{FP} \cong \overline{FR}$. $\overline{PE}$ is an altitude of $\triangle PQF$, and $\overline{PF}$ is a median of $\triangle PQR$.



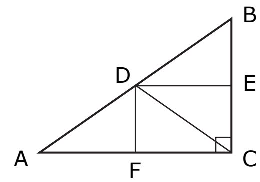
3. Explain why $QR = 4QE$.

4. Explain why $m\angle Q = 2m\angle R$.

5. In $\triangle PQF$, what type of segment is $\overline{PE}$? List all that apply.

☐ LESSON 51

$\overline{DE}$ and $\overline{DF}$ are midsegments of right $\triangle ABC$ with $\overline{AC} \perp \overline{BC}$.



1. Explain why $\overline{DE} \perp \overline{BC}$ and $\overline{DF} \perp \overline{AC}$.

2. What type of segment is $\overline{DF}$ in $\triangle ADC$? List all that apply.

3. Explain why D is the circumcenter of $\triangle ABC$. What can you say about the circumcenter of a right triangle?

☐ LESSON 52

Prove Theorem 52.1. Give a reason or reasons for each statement. (*Hint*: Note that, if $a = b + c$ and $c > 0$, then $a > b$ by the definition of greater than.)

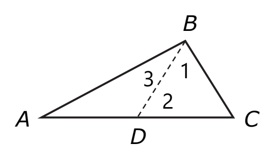
1. Given: $\triangle ABC$ with $AC > BC$

Prove: $m\angle B > m\angle A$

Because $AC > BC$, you can find D on $\overline{AC}$ such that $BC = DC$. Draw $\overline{BD}$. Then you have:

(1) $m\angle B > m\angle 1$ (2) $m\angle 2 > m\angle A$

(2) $m\angle 1 = m\angle 2$ (3) $m\angle B > m\angle A$



Do the problem(s) on the next page.

Theorem 52.2 can be proved using an indirect proof. Answer each question.

2. Given: $\triangle ABC$ with $m\angle B > m\angle A$
Prove: $AC > BC$

Assume $AC \ngtr BC$. Then $AC < BC$ or $AC = BC$.

(1) If $AC < BC$, then how do $\angle A$ and $\angle B$ compare? Explain.

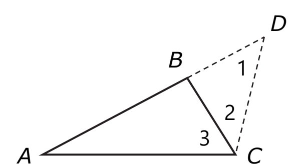
(2) If $AC = BC$, then how do $\angle A$ and $\angle B$ compare? Explain.

(3) What can you conclude from your answers above?

Prove Theorem 52.3. Give a reason or reasons for each statement.

3. Given: $\triangle ABC$
Prove: $AB + BC > AC$

Extend $\overline{AB}$ to D such that $BC = BD$. Draw $\overline{DC}$. Then you have:



(1) $m\angle ACD > m\angle 2$ (2) $m\angle ACD > m\angle 1$

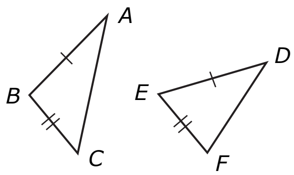
(2) $AD > AC$ (3) $AB + BC > AC$

☐ LESSON 53

Prove Theorem 53.1. Give a reason or reasons for each statement.

1. Given: $\overline{AB} \cong \overline{DE}$,
$\overline{BC} \cong \overline{EF}$,
$m\angle B > m\angle E$

Prove: $AC > DF$

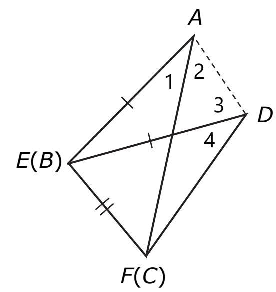


Transform $\triangle DEF$ so that $\overline{EF}$ coincides with $\overline{BC}$. Because $m\angle B > m\angle E$, $\overline{ED}$ will lie between $\overline{BA}$ and $\overline{BC}$. Draw $\overline{AD}$. Then you have:

(1) $m\angle 3 = m\angle 1 + m\angle 2$ (*Hint*: See $\triangle ABD$.)

(2) $m\angle 3 > m\angle 2$ (3) $m\angle 3 + m\angle 4 > m\angle 2$

(4) $AC > DF$ (*Hint*: See $\triangle ADF$.)



Theorem 53.2 can be proved using an indirect proof. Answer each question.

2. Given: $\overline{AB} \cong \overline{DE}$,
$\overline{BC} \cong \overline{EF}$,
$AC > DF$

Prove: $m\angle B > m\angle E$

Assume $m\angle B \ngtr m\angle E$. Then $m\angle B < m\angle E$ or $m\angle B = m\angle E$.

(1) If $m\angle B < m\angle E$, then how do AC and DF compare? Explain.

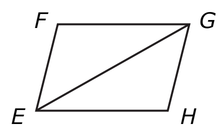
(2) If $m\angle B = m\angle E$, then how do AC and DF compare? Explain.

(3) What can you conclude from your answers above?

☐ LESSON 54

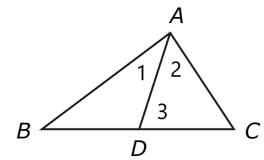
Let's try proofs involving inequalities in triangles. Complete each proof. Note that, if $a = b + c$ and $c > 0$, then $a > b$ by the definition of greater than.

1. Given: $\overline{FE} \parallel \overline{GH}$,
$\angle F \cong \angle H$
Prove: $EF + EH > EG$



STATEMENTS	REASONS
1. $\overline{FE} \parallel \overline{GH}$, $\angle F \cong \angle H$	1. Given
2. $\angle FEG \cong \angle HGE$	2.
3. $\overline{EG} \cong \overline{GE}$	3.
4. $\triangle EFG \cong \triangle GHE$	4.
5. $FG = EH$	5. CPCTC
6. $EF + FG > EG$	6.
7. $EF + EH > EG$	7. Substitution

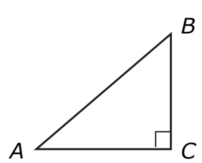
2. Given: $\overline{AD}$ bisects $\angle BAC$.
Prove: $AC > DC$



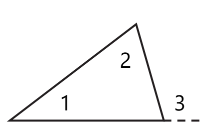
STATEMENTS	REASONS
1. $\overline{AD}$ bisects $\angle BAC$.	1. Given
2. $m\angle 1 = m\angle 2$	2.
3. $m\angle 3 = m\angle 1 + m\angle B$	3.
4. $m\angle 3 = m\angle 2 + m\angle B$	4.
5. $m\angle 3 > m\angle 2$	5. Def. of greater than
6. $AC > DC$	6.

Write a proof using any format. Problem 3 proves that the hypotenuse is always the longest side of a right triangle. Problem 4 is known as the Exterior Angle Inequality Theorem.

3. Given: $m\angle C = 90°$
Prove: $BC < AB$, $AC < AB$



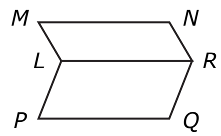
4. Given: $\angle 3$ is an exterior angle.
Prove: $m\angle 3 > m\angle 1$, $m\angle 3 > m\angle 2$



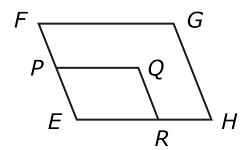
☐ LESSON 56

Write a proof using any format.

19. Given: $\square LMNR$,
$\square LPQR$
Prove: $\overline{MN} \cong \overline{PQ}$



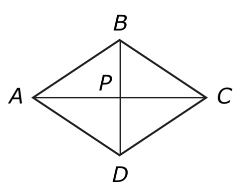
20. Given: $\square EFGH$,
$\square EPQR$
Prove: $\angle G \cong \angle Q$



☐ LESSON 58

Prove Theorems 58.1 through 58.3. (*Hint*: Use the theorems in Lesson 56.)

1. Given: rhombus $ABCD$
Prove: $\overline{AC} \perp \overline{BD}$

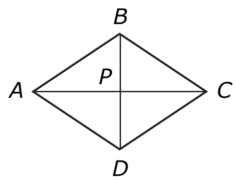


2. Given: $\square ABCD$, $\overline{AC} \perp \overline{BD}$
Prove: $ABCD$ is a rhombus.

Do the problem(s) on the next page.

3. Given: rhombus $ABCD$

Prove: $\overline{AC}$ bisects $\angle BAD$ & $\angle BCD$.
$\overline{BD}$ bisects $\angle ABC$ & $\angle ADC$.

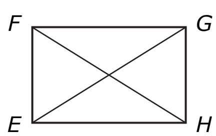


4. Given: $\square ABCD$,
$\overline{AC}$ bisects $\angle BAD$ & $\angle BCD$.

Prove: $ABCD$ is a rhombus.

5. Given: rectangle $EFGH$

Prove: $\overline{EG} \cong \overline{FH}$



6. Given: $\square EFGH$, $\overline{EG} \cong \overline{FH}$

Prove: $EFGH$ is a rectangle.

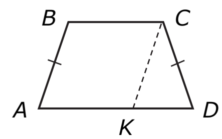
☐ LESSON 59

Read aloud the theorems in Lesson 56, then prove Theorems 59.2 and 59.3. Remember, by definition, trapezoid $ABCD$ with bases $\overline{BC}$ and $\overline{AD}$ means $\overline{BC} \parallel \overline{AD}$.

1. Given: trapezoid $ABCD$, $\overline{BA} \cong \overline{CD}$

Prove: $\angle A \cong \angle D$, $\angle B \cong \angle C$

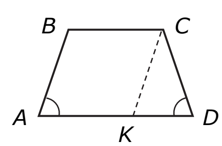
Hint: Start by drawing $\overline{CK}$ such that $\overline{BA} \parallel \overline{CK}$.



2. Given: trapezoid $ABCD$, $\angle A \cong \angle D$

Prove: $\overline{BA} \cong \overline{CD}$

Hint: Draw and use $\overline{CK}$ as you just did in Problem 1.

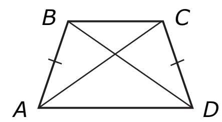


Do the problem(s) on the next page.

3. <u>Given</u>: trapezoid $ABCD$, $\overline{BA} \cong \overline{CD}$

<u>Prove</u>: $\overline{BD} \cong \overline{CA}$

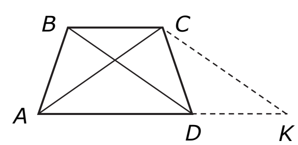
Hint: Use Thm 59.2 and congruent triangles.



4. <u>Given</u>: trapezoid $ABCD$, $\overline{BD} \cong \overline{CA}$

<u>Prove</u>: $\overline{BA} \cong \overline{CD}$

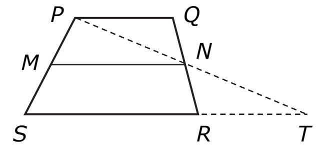
Hint: Draw $\overline{CK}$ such that $\overline{CK} \parallel \overline{BD}$ and K is on $\overleftrightarrow{AD}$.
Then you can prove $\angle K \cong \angle BDA \cong \angle CAD$.



Prove Theorem 59.1. Give a reason or reasons for each statement. Remember that you may need to give more than one reason to justify a statement.

1. Given: trapezoid $PQRS$, midsegment $\overline{MN}$
   Prove: $\overline{MN} \parallel \overline{SR} \parallel \overline{PQ}$, $MN = \frac{1}{2}(SR + PQ)$

   Draw and extend $\overline{PN}$ till it intersects $\overleftrightarrow{SR}$ at T. Then you have:

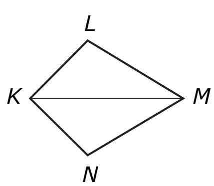
   (1) $\triangle PQN \cong \triangle TRN$

   (2) $\overline{MN}$ is a midsegment of $\triangle PST$.

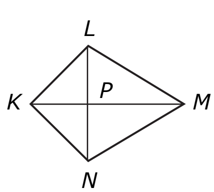
   (3) $\overline{MN} \parallel \overline{SR} \parallel \overline{PQ}$ (4) $MN = \frac{1}{2}(SR + PQ)$ (*Hint*: $PQ = RT$)

Prove Theorems 59.4 through 59.6. (*Hint*: Use congruent triangles.)

2. Given: kite $KLMN$
   Prove: $\angle L \cong \angle N$, $\angle LKM \cong \angle NKM$, $\angle LMK \cong \angle NMK$

3. Given: kite $KLMN$
   Prove: $\overline{LP} \cong \overline{NP}$, $\overline{KM} \perp \overline{LN}$

☐ LESSON 62

Identify each statement as true or false. If false, describe or draw a counterexample.

1. All squares are similar.

2. All right triangles are similar.

3. All rectangles are similar.

4. All equilateral triangles are similar.

5. All parallelograms are similar.

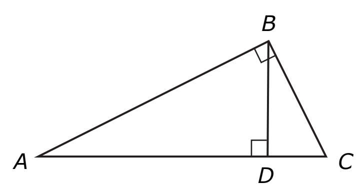
6. All acute isosceles triangles are similar.

7. All rhombuses are similar.

8. All isosceles right triangles are similar.

☐ LESSON 64

$\overline{BD}$ is the altitude to the hypotenuse of right $\triangle ABC$.

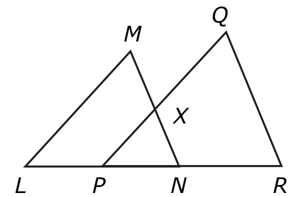


1. Explain why $\triangle ABC \sim \triangle ADB \sim \triangle BDC$.

2. Find AD if $BD = 12$ and $DC = 6$. (*Hint*: Draw and label $\triangle ADB$ and $\triangle BDC$ separately.)

☐ LESSON 65

In the diagram, $\overline{LM} \parallel \overline{PQ}$ and $\overline{MN} \parallel \overline{QR}$.



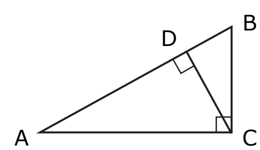
1. Explain why $\triangle LMN \sim \triangle PQR \sim \triangle PXN$.

2. Find $m\angle M$ and $m\angle Q$ if $m\angle L = 47°$ and $m\angle R = 63°$.

3. Find PN if $LM = 20$, $PQ = 25$, $LP = 8$, and $NR = 12$.

☐ LESSON 67

Read aloud Theorem 63.1, then prove Theorem 67.1.



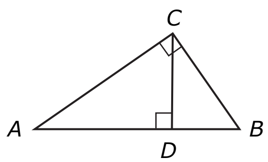
1. Given: right $\triangle ABC$ with altitude $\overline{CD}$ to hypotenuse $\overline{AB}$
   Prove: $\triangle ABC \sim \triangle ACD \sim \triangle CBD$

☐ LESSON 68

Read aloud Theorem 67.1, then prove Theorems 68.1 and 68.2. (*Hint*: Use proportional sides of similar triangles.)

1. Given: right $\triangle ABC$
with altitude $\overline{CD}$
to hypotenuse $\overline{AB}$

Prove: $CD^2 = AD \cdot BD$



2. Given: right $\triangle ABC$
with altitude $\overline{CD}$
to hypotenuse $\overline{AB}$

Prove: $AC^2 = AD \cdot AB$,
$BC^2 = BD \cdot BA$

☐ LESSON 69

Prove two properties of proportions.

1. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Hint: Add 1 to both sides.

2. If $\frac{a+b}{b} = \frac{c+d}{d}$, then $\frac{a}{b} = \frac{c}{d}$.

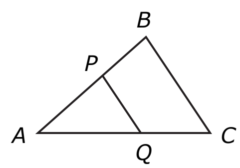
Hint: Do the opposite of Problem 1.

Do the problem(s) on the next page.

Prove Theorem 69.1. Remember, to prove an if and only if statement, you must prove both directions of the statement. (*Hint*: Use the two properties of proportions that you just proved.)

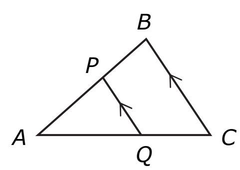
3. Given: $\frac{PB}{AP} = \frac{QC}{AQ}$

Prove: $\overline{PQ} \parallel \overline{BC}$

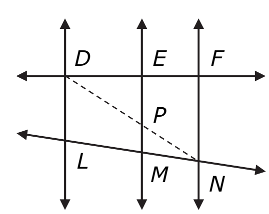


4. Given: $\overline{PQ} \parallel \overline{BC}$

Prove: $\frac{PB}{AP} = \frac{QC}{AQ}$

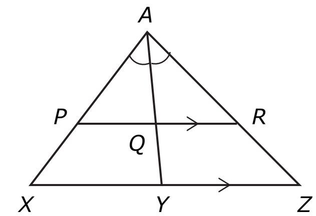


5. Theorem 69.2 can be proved using Theorem 69.1 after an auxiliary line is drawn. Use the diagram to write Given and Prove statements. Then write a proof. (*Hint*: Notice that you can apply Theorem 69.1 to $\triangle DNF$ and $\triangle DNL$ respectively.)



☐ LESSON 70

$\overline{PR} \parallel \overline{XZ}$ and $\overline{AY}$ bisects $\angle XAZ$. Complete each statement.



1. $\dfrac{AP}{PX} = \dfrac{AQ}{?} = \dfrac{?}{RZ}$

2. $\dfrac{AP}{AX} = \dfrac{PQ}{?} = \dfrac{?}{XZ} = \dfrac{AR}{?}$

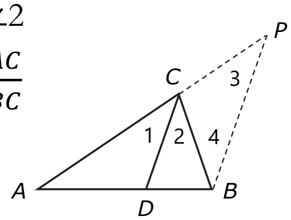
3. $\dfrac{XY}{YZ} = \dfrac{AX}{?}$

4. $\dfrac{AP}{PQ} = \dfrac{?}{RQ}$

Prove Theorem 70.1. Give a reason or reasons for each statement. Remember that you may need to give more than one reason to justify a statement.

5. Given: $\angle 1 \cong \angle 2$

Prove: $\dfrac{AD}{BD} = \dfrac{AC}{BC}$



Draw a line through B parallel to $\overline{DC}$. Extend $\overline{AC}$ to intersect the line at P. Then you have:

(1) $\angle 1 \cong \angle 3$ (2) $\angle 2 \cong \angle 4$ (3) $\angle 3 \cong \angle 4$

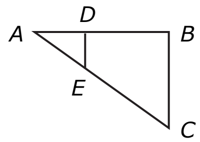
(4) $BC = PC$ (5) $\dfrac{AD}{BD} = \dfrac{AC}{PC}$ (6) $\dfrac{AD}{BD} = \dfrac{AC}{BC}$

☐ LESSON 71

Write a proof using any format.

1. Given: $AB = 3AD$, $AC = 3AE$

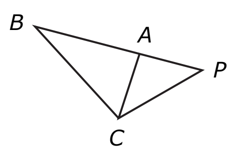
   Prove: $\triangle ABC \sim \triangle ADE$



Hint: Which similarity criterion can be used?

2. Given: $\angle B \cong \angle PCA$

   Prove: $PC^2 = PA \cdot PB$

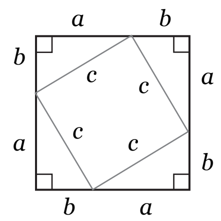


Hint: Which two triangles are similar?

☐ LESSON 74

Prove the Pythagorean Theorem using a square.

1.



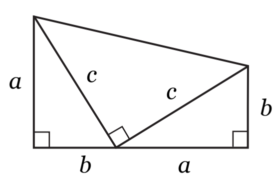
a. Find the area of the large square by squaring its side length.

b. Find the area of the large square by adding the areas of the four right triangles and the small square.

c. Set the two areas equal and simplify the equation to prove the theorem.

Prove the Pythagorean Theorem using a trapezoid.

2.



a. The area of a trapezoid is half the product of the height and the sum of the two bases. Find the area of the trapezoid.

b. Find the area of the trapezoid by adding the areas of the three right triangles.

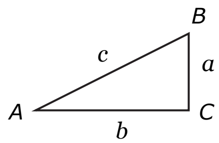
c. Set the two areas equal and simplify the equation to prove the theorem.

☐ LESSON 75

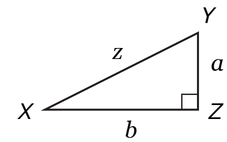
Prove Theorems 75.1. Give a reason or reasons for each statement.

1. Given: $\triangle ABC$ with $c^2 = a^2 + b^2$ and longest side c

Prove: $\triangle ABC$ is a right triangle.



Draw a right $\triangle XYZ$ with legs a and b and with hypotenuse z. Then you have:



(1) $z^2 = a^2 + b^2$ (2) $z^2 = c^2$

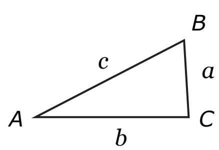
(3) $z = c$ (4) $\triangle ABC \cong \triangle XYZ$

Congruent triangles have congruent angles, so $\triangle ABC$ also has a right angle. This proves that $\triangle ABC$ is a right triangle.

Theorems 75.2 and 75.3 can be proved in a similar manner. Write a proof. (*Hint*: Draw a right $\triangle XYZ$ as above, then use the Converse of Hinge Theorem [53.2] to compare $\angle C$ and $\angle Z$.)

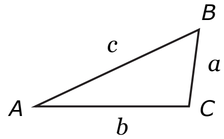
2. Given: $\triangle ABC$ with $c^2 < a^2 + b^2$

Prove: $\triangle ABC$ is an acute triangle.



3. Given: $\triangle ABC$ with $c^2 > a^2 + b^2$

Prove: $\triangle ABC$ is an obtuse triangle.



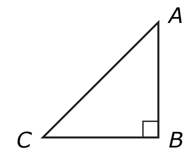
☐ LESSON 76

Read aloud the theorems in Lesson 40, then prove Theorems 76.1 and 76.2.

1. Given: $\triangle ABC$ is a 45°-45°-90° triangle.

Prove: $AB = BC$, $AC = AB\sqrt{2}$

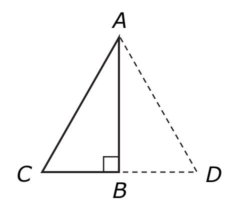
Hint: Use the Base Angles Converse [40.2] to prove $AB = BC$ first. Then use the Pythagorean Theorem to find AC.



2. Given: $\triangle ABC$ is a 30°-60°-90° triangle.

Prove: $AC = 2BC$, $AB = BC\sqrt{3}$

Hint: Construct $\triangle ABD$ such that $BC = BD$. First prove that $\triangle ACD$ is equilateral and thus $AC = 2BC$. Then use the Pythagorean Theorem to find AB.



☐ LESSON 77

Use special right triangles to find the indicated measure(s) in simplest radical form. Be sure you know the terms.

1. The shorter leg of a **30°-60°-90° triangle** is 6 cm long. Find the perimeter.

2. A **leg** of an **isosceles right triangle** is 5 in long. Find the perimeter.

3. The perimeter of a **square** is 36 in. Find the length of a **diagonal**.

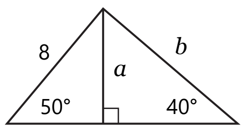
4. The side length of an **equilateral triangle** is 8 cm. Find the length of an **altitude**.

5. In an **isosceles triangle** with **base angle** 30°, the **median** from the **vertex angle** to the **base** is 40 mm long. Find the perimeter.

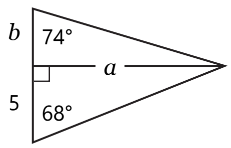
☐ LESSON 78

Find the values of the variables. Round all calculations to the nearest tenth.

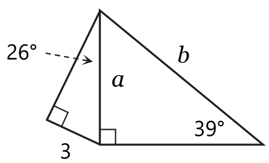
1.



2.



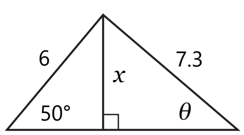
3.



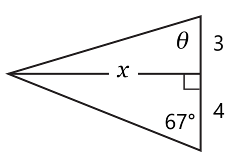
☐ LESSON 79

Find side x to the nearest tenth, then find angle θ to the nearest tenth of a degree.

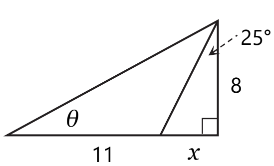
1.



2.



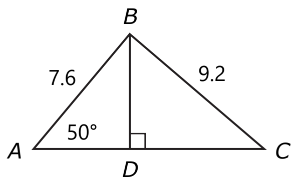
3.



☐ **LESSON 80**

Let's try problems involving multiple right triangles. Study the example first.

➔ **EXAMPLE** Solve $\triangle ABD$ and $\triangle BCD$. Round all calculations to the nearest tenth.

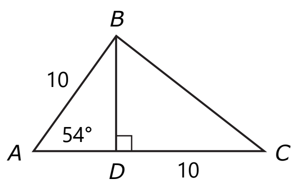


1. Solve $\triangle ABD$ using AB and $\angle A$.
   BD = 7.6 sin 50° ≈ 5.8
   AD = 7.6 cos 50° ≈ 4.9
   $m\angle ABD$ = 90° − 50° = 40°

2. Solve $\triangle BCD$ using BC and BD.
   $m\angle C$ ≈ $\sin^{-1}$ (5.8/9.2) ≈ 39.1°
   $m\angle CBD$ ≈ 90° − 39.1° = 50.9°
   DC ≈ 9.2 sin 50.9° ≈ 7.1
   (or DC ≈ 9.2 cos 39.1° ≈7.1)

Solve each triangle. Round all calculations to the nearest tenth.

1.

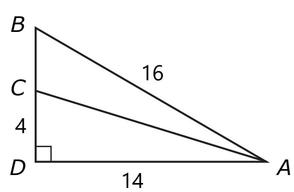


a. $\triangle ABD$

b. $\triangle BCD$

c. $\triangle ABC$

2.



a. $\triangle ACD$

b. $\triangle ABD$

c. $\triangle ABC$

☐ LESSON 81

Find the indicated measure(s) to the nearest tenth.

1. An equilateral triangle has side length 6. What is the height? What is the area?

2. A rhombus has perimeter 16, and one angle is 40°. What is the side length? What is the area?

3. A parallelogram has perimeter 30. One side is 10, and one angle is 110°. What is the area?

4. A kite has side lengths of 5 and 7. Its two congruent non-vertex angles measure 106°. What is the perimeter? What is the area?

☐ **LESSON 82**

Solve. Round all calculations to the nearest tenth.

1. A kite with a string 80 feet long makes an angle of between 40° and 65° with the ground. What are the minimum and maximum heights of the kite?

2. An airplane takes off at an angle of elevation of 22° and continues to ascend at the same angle with an average rate of 100 ft/s. How high is the plane above the ground after 5 seconds?

☐ **LESSON 84**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

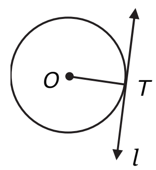
1. Line $y = x$ is reflected over the y-axis. Write an equation of the image in slope-intercept form.

2. Line l passes through (0, 2) and (2, 0). Line q is the image of line l after a reflection over the x-axis. What is the intersection of the two lines?

☐ LESSON 91

Prove Theorems 91.1 and 91.2.

1. Given: Line l is tangent to $\odot O$ at point T.
Prove: $l \perp \overline{OT}$

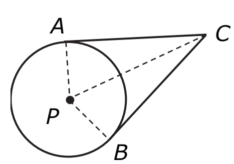


Hint: Use an indirect proof. Assume that l is not perpendicular to $\overline{OT}$. Then there must be a point K on l such that $l \perp \overline{OK}$. Compare the lengths between $\overline{OT}$ and $\overline{OK}$. Show that $OT < OK$ and $OT > OK$ are true respectively, which is a contradiction.

2. Given: $l \perp \overline{OT}$
Prove: Line l is tangent to $\odot O$.

Hint: Use an indirect proof. Assume that l is not tangent to $\odot O$. Then l must intersect $\odot O$ at another point K besides T. Prove $OT = OK$ and $OT < OK$ respectively to lead to a contradiction.

3. Given: $\overline{CA}$ and $\overline{CB}$ are tangents to $\odot P$.
Prove: $\overline{CA} \cong \overline{CB}$



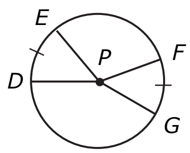
Hint: Draw $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. Notice that $\triangle CAP$ and $\triangle CBP$ are right triangles. Prove $\triangle CAP \cong \triangle CBP$, then you can say $\overline{CA} \cong \overline{CB}$.

☐ **LESSON 92**

Prove Theorem 92.1. (*Hint*: Use the definitions of arc measure and congruent arcs.)

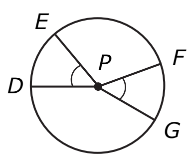
1. Given: $\widehat{DE} \cong \widehat{FG}$

Prove: $\angle DPE \cong \angle FPG$



2. Given: $\angle DPE \cong \angle FPG$

Prove: $\widehat{DE} \cong \widehat{FG}$

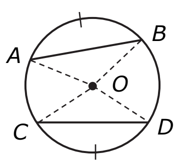


☐ **LESSON 93**

Prove Theorems 93.1 and 93.2. (*Hint*: Use congruent triangles formed by auxiliary lines.)

1. Given: $\widehat{AB} \cong \widehat{CD}$

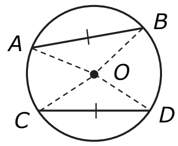
Prove: $\overline{AB} \cong \overline{CD}$



Do the problem(s) on the next page.

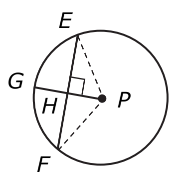
2. Given: $\overline{AB} \cong \overline{CD}$

Prove: $\widehat{AB} \cong \widehat{CD}$



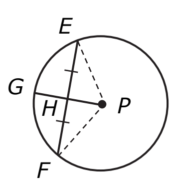
3. Given: $\overline{PG} \perp \overline{EF}$

Prove: $\overline{HE} \cong \overline{HF}$, $\widehat{GE} \cong \widehat{GF}$



4. Given: $\overline{HE} \cong \overline{HF}$

Prove: $\overline{PG} \perp \overline{EF}$

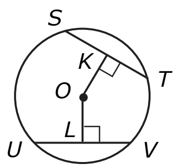


☐ **LESSON 94**

Prove Theorem 94.1. (*Hint*: Draw auxiliary lines to form right triangles.)

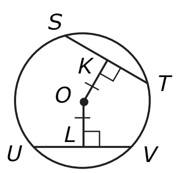
1. Given: $\overline{OK} \perp \overline{ST}$, $\overline{OL} \perp \overline{UV}$, $\overline{ST} \cong \overline{UV}$

Prove: $\overline{OK} \cong \overline{OL}$



2. Given: $\overline{OK} \perp \overline{ST}$, $\overline{OL} \perp \overline{UV}$, $\overline{OK} \cong \overline{OL}$

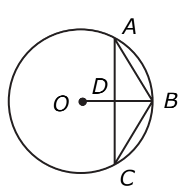
Prove: $\overline{ST} \cong \overline{UV}$



Write a proof using any format.

3. Given: $\overline{AD} \cong \overline{DC}$

Prove: $\overline{AB} \cong \overline{BC}$

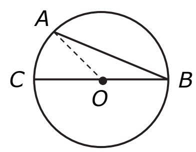


☐ **LESSON 95**

To prove Theorem 95.1, you must prove three cases. Write a proof for each case. (*Hint*: Prove the first case, then use it to prove the second and third cases.)

1. Given: inscribed $\angle ABC$,
diameter $\overline{BC}$

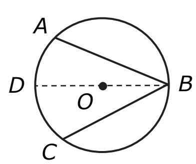
Prove: $m\angle B = m\widehat{AC}/2$
or $m\widehat{AC} = 2m\angle B$



Hint: Show that $\triangle OAB$ is isosceles. Then use the Base Angles Theorem [40.1] and the Triangle Exterior Angle Theorem [32.2] to prove that $m\angle AOC$ is twice $m\angle B$.

2. Given: inscribed $\angle ABC$,
diameter $\overline{DB}$

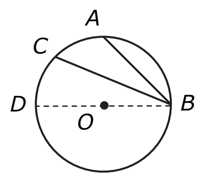
Prove: $m\angle B = m\widehat{AC}/2$
or $m\widehat{AC} = 2m\angle B$



Do the problem(s) on the next page.

3. <u>Given</u>: inscribed $\angle ABC$,
diameter $\overline{DB}$

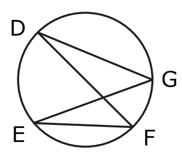
<u>Prove</u>: $m\angle B = m\widehat{AC}/2$
or $m\widehat{AC} = 2m\angle B$



Theorem 95.2 can be proved using Theorem 95.1. Write a proof for each case.

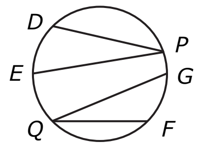
4. <u>Given</u>: inscribed $\angle DGE$,
inscribed $\angle DFE$

<u>Prove</u>: $\angle G \cong \angle F$



5. <u>Given</u>: inscribed $\angle DPE$,
inscribed $\angle FQG$,
$\widehat{DE} \cong \widehat{FG}$

<u>Prove</u>: $\angle P \cong \angle Q$

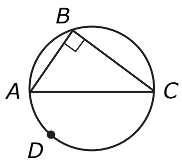


☐ **LESSON 96**

Prove Theorem 96.1. (*Hint*: What is $m\widehat{ADC}$?)

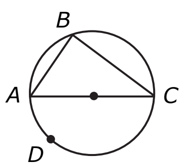
1. Given: inscribed $\triangle ABC$,
$m\angle B = 90°$

Prove: $\overline{AC}$ is a diameter.



2. Given: inscribed $\triangle ABC$,
diameter $\overline{AC}$

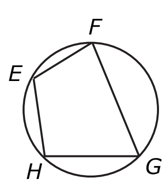
Prove: $m\angle B = 90°$



Complete the proof of the first statement of Theorem 96.2. Notice that you can apply the same reasoning to prove the second statement of the theorem.

3. Given: inscribed $EFGH$

Prove: $m\angle E + m\angle G = 180°$,
$m\angle F + m\angle H = 180°$



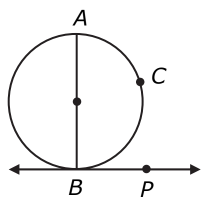
STATEMENTS	REASONS
1. $EFGH$ is inscribed in a circle.	1. Given
2. $m\widehat{FGH} = 2m\angle E$ $m\widehat{FEH} = 2m\angle G$	2.
3. $m\widehat{FGH} + m\widehat{FEH} = 360°$	3. A circle measures 360°.
4. $2m\angle E + 2m\angle G = 360°$	4.
5. $m\angle E + m\angle G = 180°$	5.

☐ **LESSON 97**

To prove Theorem 97.1, you must prove three cases. Write a proof for each case. (*Hint*: Prove the first case, then use it to prove the second and third cases.)

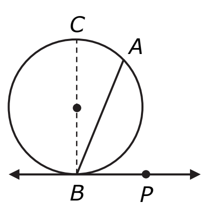
1. Given: tangent $\overleftrightarrow{BP}$, diameter $\overline{AB}$

   Prove: $m\angle ABP = \frac{1}{2} m\widehat{ACB}$



2. Given: tangent $\overleftrightarrow{BP}$, acute $\angle ABP$

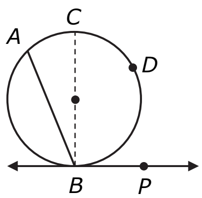
   Prove: $m\angle ABP = \frac{1}{2} m\widehat{AB}$



Do the problem(s) on the next page.

3. Given: tangent $\overleftrightarrow{BP}$, obtuse $\angle ABP$

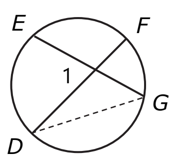
Prove: $m\angle ABP = \frac{1}{2} m\widehat{ACB}$



Read aloud Theorems 95.1 and 32.2, then prove Theorem 97.2.

4. Given: chords $\overline{DF}$ and $\overline{EG}$

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{DE} + m\widehat{FG})$



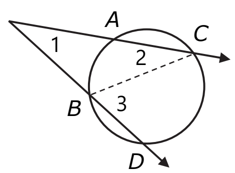
Hint: Notice that $\angle D$ and $\angle G$ are inscribed angles and that $\angle 1$ is an exterior angle of the triangle formed by $\angle D$ and $\angle G$.

☐ LESSON 98

Prove each case of Theorem 98.1. (*Hint*: ∠3 is an exterior angle of each triangle.)

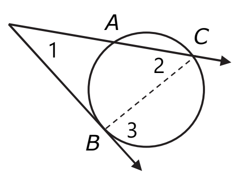
1. Given: two secants

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{CD} - m\widehat{AB})$



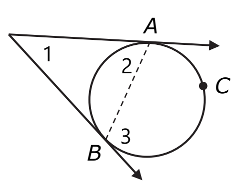
2. Given: a secant, a tangent

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{CB} - m\widehat{AB})$



3. Given: two tangents

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{ACB} - m\widehat{AB})$

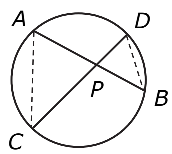


☐ **LESSON 99**

Prove Theorems 99.1 through 99.3. (*Hint*: Use proportional sides of similar triangles.)

1. Given: two chords

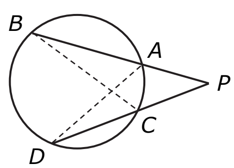
Prove: $PA \cdot PB = PC \cdot PD$



Hint: Prove $\triangle PAC \sim \triangle PDB$.

2. Given: two secants

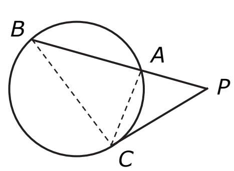
Prove: $PA \cdot PB = PC \cdot PD$



Hint: Prove $\triangle PBC \sim \triangle PDA$.

3. Given: a tangent, a secant

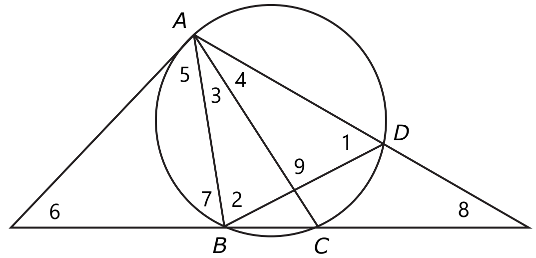
Prove: $PA \cdot PB = PC^2$



Hint: Prove $\triangle PAC \sim \triangle PCB$.

Let's try some problems that will require all of your knowledge on angle relationships in circles.

➔ **EXAMPLE** $m\widehat{AB} = 2x + 12, m\widehat{BC} = x, m\widehat{CD} = x + 4,$ and $m\widehat{AD} = 3x - 6$. Find all the arc measures, then find all the numbered angle measures



Find all arc measures:

$2x + 12 + x + x + 4 + 3x - 6 = 360; x = 50$

$m\widehat{AB} = 112°, m\widehat{BC} = 50°, m\widehat{CD} = 54°, m\widehat{AD} = 144°$

Find all numbered angle measures:

∠1 is an inscribed angle, so $m\angle 1 = m\widehat{AB}/2 = 56°$.

∠2 is an inscribed angle, so $m\angle 2 = m\widehat{AD}/2 = 72°$.

∠3 is an inscribed angle, so $m\angle 3 = m\widehat{BC}/2 = 25°$.

∠4 is an inscribed angle, so $m\angle 4 = m\widehat{CD}/2 = 27°$.

∠5 is formed by a chord and a tangent, so $m\angle 5 = m\widehat{AB}/2 = 56°$.

∠6 is formed by a tangent and a secant, so $m\angle 6 = (m\widehat{ADC} - m\widehat{AB})/2 = 43°$.

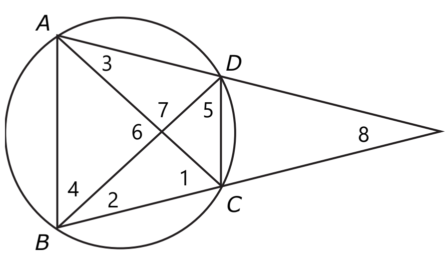
∠7, ∠5, and ∠6 forms a triangles, so $m\angle 7 = 180 - m\angle 5 - m\angle 6 = 81°$.

∠8 is formed by two secants, so $m\angle 8 = (m\widehat{AB} - m\widehat{CD})/2 = 29°$.

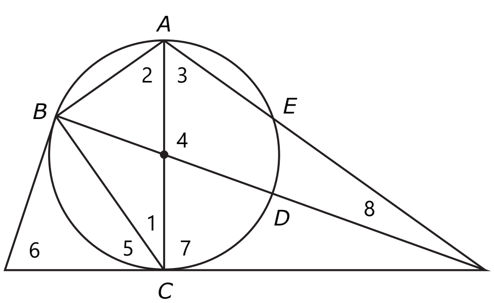
∠9 is formed by two chords, so $m\angle 9 = (m\widehat{BC} + m\widehat{AD})/2 = 97°$.

Use the given information to find all the arc measures and numbered angle measures.

1. $\overline{AD} \cong \overline{BC}, m\widehat{AB} = 112°, m\widehat{CD} = 56°$



2. $m\widehat{AB} = m\widehat{CD} = 70°, m\widehat{DE} = 38°$

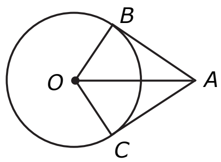


☐ **LESSON 101**

Write a proof using any format.

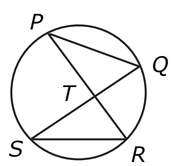
1. Given: tangent $\overline{AB}$, tangent $\overline{AC}$

Prove: $\angle OAB \cong \angle OAC$



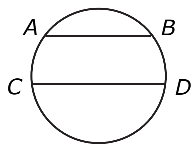
2. Given: $\widehat{PQ} \cong \widehat{SR}$

Prove: $\triangle PQT \cong \triangle SRT$



3. Given: $\overline{AB} \parallel \overline{CD}$

Prove: $\widehat{AC} \cong \widehat{BD}$



Hint: Draw $\overline{BC}$.

☐ LESSON 106

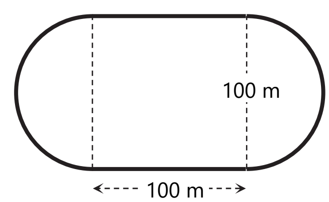
Solve. Use 22/7 as π. Round all calculations to the nearest whole number.

1. The diameter of the wheel of a car is 80 cm. How far will the car travel in 5,000 revolutions? Give your answer in km. (*Hint*: Recall that 1 km = 1,000 m = 100,000 cm.)

2. The diameter of a wheel is 75 cm. How many revolutions will the wheel make to travel 2 km?

A running track is shaped like a rectangle with semicircles at the two ends.

3. Find the length of the track. Use 22/7 as an approximation for π. Round to the nearest meter.



4. How many minutes will it take a runner to run around the track four times if the runner runs at an average speed of 2 meters per second? Round to the nearest minute.

☐ LESSON 113

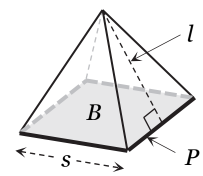
Understand the proportional relationship between arc length and sector area in a circle. Use this relationship to find the indicated measure. Leave π in your answer.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{arc length}}{\text{circumference}} = \frac{\text{sector area}}{\text{circle area}}$$

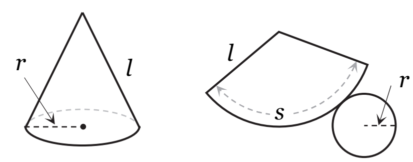
1. What is the area of a sector with radius 9 and arc length 6π?

Derive the lateral area formulas yourself.

2. Derive the lateral area formula for a regular pyramid, $LA = \frac{1}{2}Pl$. First, find the lateral area of a pyramid whose base is a regular n-gon with side length s and whose slant height is l. Then, use base perimeter $P = ns$ to get the first formula on the right.



3. Derive the lateral area formula for a right cone, $LA = \pi r l$. First, find arc length s of the sector in terms of r. Then, use a proportion to find the area of the sector in terms of r and l. The area of the sector is the lateral area of the cone.



☐ LESSON 115

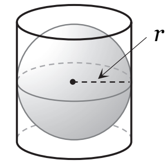
1. A right cone has radius 7 cm and surface area 224π cm^2. What is the volume?

☐ **LESSON 116**

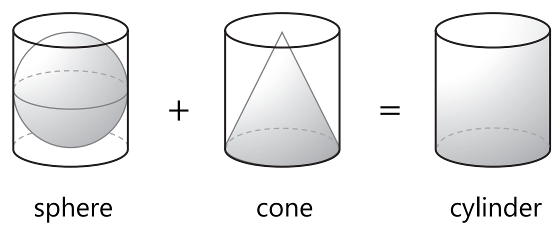
Derive the surface area and volume formulas for a sphere.

1. The great mathematician Archimedes proved that the surface area of a sphere equals the lateral area of the cylinder that circumscribes it. This means that the surface area formula for a sphere can be derived from the lateral area formula for a cylinder.

   Derive the surface area formula yourself. Use the diagram on the right. First, find the radius and height of a cylinder that circumscribes a sphere with radius r. Then, find the lateral area of the cylinder in terms of r.



2. Archimedes also proved that the volumes of a sphere, a cone, and a cylinder are related as shown on the right. Use this relationship to derive the volume formula for a sphere.



☐ **LESSON 122**

1. On a coordinate map, Morgan's house is located at (0, 4), and the library is at (4, 7). Morgan rode her bicycle from her house to the library at an average speed of 10 km/hr. How long did it take? One unit on the map equals 1 km.

☐ **LESSON 125**

Find the distance between each pair of parallel lines in simplest radical form.

1. Follow the steps to find the distance between $2x + y = 3$ and $2x + y = 8$.

a. Find the y-intercept of $2x + y = 3$.

b. Find the line perpendicular to $2x + y = 8$ passing through the y-intercept.

c. Find the intersection between $2x + y = 8$ and the perpendicular line.

d. Find the distance between the y-intercept and the intersection.

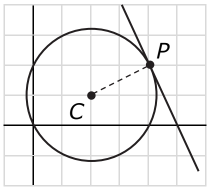
2. Find the distance between $y = -x + 2$ and $x + y = 6$.

3. Find the distance between $y = 2x - 1$ and $2x - y = 6$.

☐ **LESSON 128**

Let's try problems involving equations of tangent lines.

1. Recall that a tangent and a radius are perpendicular at the point of tangency. A circle centered at $C(2, 1)$ passes through $P(4, 2)$. Let's find an equation of the tangent line at P.

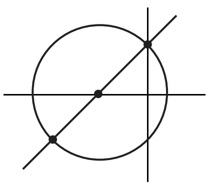


   a. What is the slope of $\overline{CP}$?

   b. What is the slope of the line tangent to circle C at P?

   c. What is the point-slope equation of the line tangent to circle C at P?

2. Recall that you can find the intersection(s) of two graphs by solving the system of their equations. Let's find where line $x - y = -2$ and circle $(x + 2)^2 + y^2 = 8$ intersect.



   a. Solve the line equation for y.

   b. Substitute the value for y into the circle equation and solve for x.

   c. Use the values of x to find the corresponding values of y.

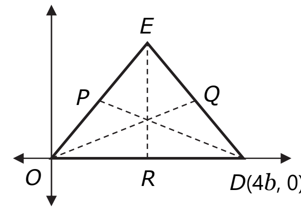
3. Write the point-slope equation of a line tangent to a circle with center $C(0, 1)$ at $P(3, 0)$.

4. Find the point(s) of intersection between line $y = -x + 4$ and circle $(x - 1)^2 + (y - 2)^2 = 13$.

☐ **LESSON 130**

Isosceles $\triangle ODE$ with base $4b$ and height $2h$ is placed as shown.

1. The base is $\overline{OD}$. What are the coordinates of E?



2. Find midpoints P, Q, and R of the sides respectively.

3. Show that $\overline{ER} \perp \overline{OD}$. This proves that the median to the base of an isosceles triangle is perpendicular to the base.

4. Show that $\overline{OQ} \cong \overline{DP}$. This proves that the medians drawn to the congruent sides of an isosceles triangle are congruent.

Redo the proof you studied earlier. Write a coordinate proof yourself.

5. The midpoint of the hypotenuse of a right triangle is equidistant from each of the vertices. (*Hint*: Start by placing a right triangle with legs $2a$ and $2b$ on a coordinate plane.)

☐ LESSON 136

You can now construct a triangle congruent to a given triangle. Do your construction(s) in your notebook or on a separate sheet of paper.

1. Draw a large acute $\triangle ABC$. Construct $\triangle XYZ$ congruent to $\triangle ABC$ by constructing $\overline{XY}$ congruent to $\overline{AB}$, $\angle X$ congruent to $\angle A$, and then $\angle Y$ congruent to $\angle B$.

2. Use your ruler and protractor to check that all corresponding sides and angles of the two triangles are congruent.

3. What congruence criterion is used in this construction?

☐ LESSON 137

You can now construct the circumcenter of a triangle. Do your construction(s) in your notebook or on a separate sheet of paper.

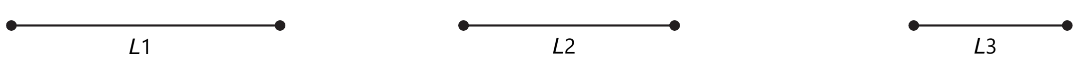
1. Draw a large acute $\triangle ABC$. Construct the circumcenter of $\triangle ABC$. (*Hint*: Recall that the circumcenter is where the perpendicular bisectors of the sides meet.)

2. Use your ruler to check that the circumcenter is equidistant from all three vertices.

3. Draw a large right $\triangle DEF$ using your straightedge and protractor. Construct the circumcenter of $\triangle DEF$. Where is the circumcenter of your triangle?

☐ LESSON 138

You can now construct rectangles given side lengths. Do your construction(s) in your notebook or on a separate sheet of paper.



1. Construct a rectangle with base length $L1$ and height $L2$. (*Hint*: Construct a copy of segment $L1$, construct a perpendicular line at each end, then construct a copy of segment $L2$ on each line.)

2. Construct a rectangle with base length $L2$ and height $L3$.

☐ LESSON 139

Do your construction(s) in your notebook or on a separate sheet of paper.

1. Construct a 30°-60°-90° triangle. (*Hint*: It is half an equilateral triangle.)

☐ LESSON 140

Do your construction(s) in your notebook or on a separate sheet of paper.

1. Draw a large circle. Construct a not-tilted square inscribed in your circle. (*Hint*: Construct two perpendicular diameters and the bisectors of the angles between them. The 4 points where the circle intersects the angle bisectors are the vertices of your square.)

☐ LESSON 141

All four centers of an equilateral triangle coincide. Let's see if it is true. Do your construction(s) in your notebook or on a separate sheet of paper.

1. Draw a segment of any length. Construct an equilateral triangle with the segment as a side.

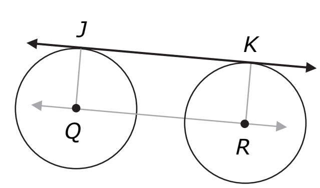
2. Construct one angle bisector of the triangle. Extend it to intersect the opposite side. If your construction is correct, this bisector should also be a median, an altitude, and a perpendicular bisector. Use a ruler and a protractor to check your work.

3. Construct the other two angle bisectors to find the incenter. Because the angle bisectors are also the medians, altitudes, and perpendicular bisectors, the incenter is also the centroid, orthocenter, and circumcenter. If your construction is correct, the incenter should be equidistant from the vertices and divide the medians in the ratio 2:1. Check your work.

☐ LESSON 142

Do your construction(s) in your notebook or on a separate sheet of paper.

1. Let's construct a common tangent to two congruent circles. Draw two congruent circles Q and R such that they do not overlap. Draw $\overleftrightarrow{QR}$. Construct radii $\overline{QJ}$ and $\overline{RK}$ perpendicular to $\overleftrightarrow{QR}$. Draw $\overleftrightarrow{JK}$, then measure $\angle QJK$ and $\angle RKJ$. Your angles should be 90°. Explain why. (*Hint*: First show that $QRKJ$ is a parallelogram.)



☐ **LESSON 146**

A 2-digit number is formed from the digits 1 to 9 with repetition allowed.

1. What is the probability that the number is 23? (*Hint*: This is the same as asking for the probability that the first digit is 2 and the second digit is 3.)

2. What is the probability that the number is 88?

3. What is the probability that the number is less than 40?

4. What is the probability that the number is divisible by 5?

☐ **LESSON 147**

A 2-digit number is formed from the digits 1 to 9 without repetition allowed.

1. What is the probability that the number is 23?

2. What is the probability that the number is 88?

3. What is the probability that the number is less than 40?

4. What is the probability that the number is divisible by 5?

☐ LESSON 148

A 2-digit number is formed from the digits 1 to 9 with repetition allowed.

1. Use the counting principle to find the probability that the number is less than 50.

2. Use the counting principle to find the probability that the number is odd.

3. Use the multiplication rule to find the probability that the number is less than 50 and odd. (*Hint*: What is the probability that the first digit is less than 5 and the second digit is odd?)

4. Use the addition rule to find the probability that the number is less than 50 or odd.

☐ LESSON 149

Find each probability.

1. A die is rolled five times. What is the probability of rolling all odd numbers?

2. x and y are chosen randomly from 1 to 9. What is the probability that the product xy is odd?

3. A 2-digit number is formed from the digits 0 to 9 with repetition allowed. What is the probability that the number is divisible by 5? (*Hint*: The zero cannot be in the tens place.)

4. A bag contains x red, $2x$ white, and y black balls. The probability of drawing a black ball is $2/5$. What is the probability of drawing a red ball?

☐ **LESSON 150**

40 students are surveyed. 7 of them play both tennis and basketball, and 6 play neither. There are twice as many students who play tennis only as those who play basketball only.

1. Draw a Venn diagram to represent the survey results.

2. How many students play tennis only?

3. What is the probability that a randomly selected student does not play tennis?

4. What is the probability that a randomly selected student plays tennis or basketball?

☐ **LESSON 152**

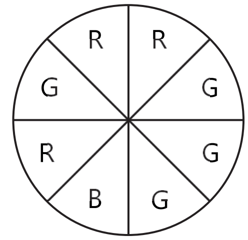
25 juniors and 25 seniors in a high school are surveyed about whether they prefer to drink juice or soda. 12 juniors answered juice while 16 seniors answered soda.

1. Construct a two-way table for the survey results.

2. Determine if "being a junior" and "preferring juice" are independent. (*Hint*: Find necessary probabilities first.)

☐ **LESSON 154**

A game is played by throwing a dart at a dartboard. A player loses 20 points if the dart lands on R, gains 20 points if it lands on G, and gains 12 points if it lands on B. Let X be the number of points the player gets.



1. Make a probability distribution table for X. (*Hint*: The possible values of X are -20, 20, and 12.)

2. Find the expected value of X. Is this game favorable to the player? Would you play? Why or why not?

☐ **LESSON 157**

A club of 5 boys and 4 girls elects a president, a vice-president, and a treasurer.

1. What is the probability that all three positions are filled by boys?

2. What is the probability that only the president is a girl?

3. What is the probability that at least one position is filled by a boy?

☐ LESSON 159

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. Explain why a quadrilateral with four congruent angles must be a rectangle. (*Hint*: Use the angle sum property.)

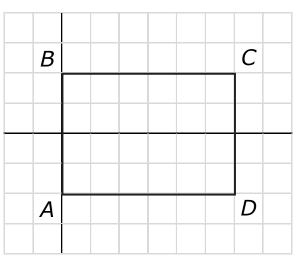
2. Write the formula that gives the measure of one interior angle of a regular polygon with n sides.

3. Write the formula that gives the measure of one exterior angle of a regular polygon with n sides.

☐ LESSON 160

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. Which transformation maps the rectangle onto itself? Select all that apply.



A) a reflection over the x-axis

B) a reflection over the line $x = 3$

C) a reflection over the line $y = 3$

D) a rotation of 180° about the origin

E) a rotation of 180° about point (3, 0)

☐ **LESSON 161**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. Write a paragraph proof of the Polygon Interior Angles Theorem [32.3].

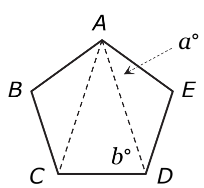
Given: a polygon with n sides

Prove: interior angle sum = $180(n - 2)$

☐ **LESSON 162**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

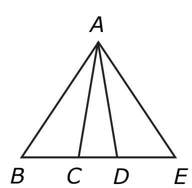
1. $ABCDE$ is a regular pentagon. Find the values of a and b. (*Hint*: What is the measure of an interior angle of a regular pentagon?)



2. Complete the proof.

Given: $\triangle ACD$ is isosceles.
$\overline{BD} \cong \overline{CE}$

Prove: $\triangle ABE$ is isosceles.



STATEMENTS	REASONS
1. $\triangle ACD$ is isosceles.	1. Given
2. $\overline{AC} \cong \overline{AD}$	2. Definition of isosceles triangle
3. $\angle ACE \cong \angle ADB$	3.
4. $\overline{BD} \cong \overline{CE}$	4. Given
5. $\triangle ABD \cong \triangle AEC$	5.
6. $\overline{AB} \cong \overline{AE}$	6.
7. $\triangle ABE$ is isosceles.	7.

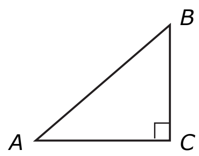
☐ LESSON 163

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. Prove that the hypotenuse is always the longest side of a right triangle.

   Given: $m\angle C = 90°$

   Prove: $AB > BC$, $AB > AC$



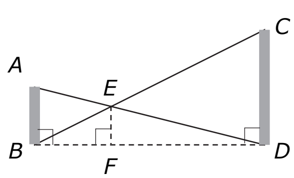
☐ LESSON 164

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. List three ways to prove a quadrilateral is a parallelogram using sides. (*Hint*: Don't forget the definition of parallelogram.)

☐ LESSON 165

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.



1. Two pillars with heights 12 m and 24 m are connected with wires. The pillars are 48 m apart. How high is the intersection above the ground? (*Hint*: Let $EF = x$ and $BF = y$. Then you can set up two equations.)

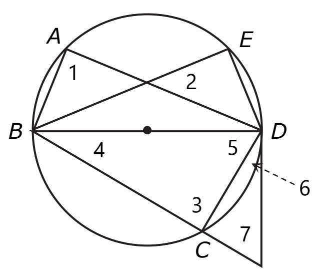
☐ **LESSON 166**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. A plane flying at an altitude of 50,000 feet descends to an altitude of 10,000 feet over 10 ground miles. There are 5,280 feet in a mile. What is the angle of depression to the nearest degree?

☐ **LESSON 167**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.



1. In the diagram, $m\widehat{AB} = m\widehat{DE} = 46°$ and $m\widehat{CD} = 60°$. Find all the arc measures, then find all the numbered angle measures.

☐ **LESSON 168**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. What is the degree measure of the acute angle formed by the hands of a clock at 2:00?

☐ **LESSON 169**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. What is the radius of a sphere whose surface area and volume are equal?

2. A right cylinder with radius 5 cm and height 6 cm is cut perpendicular to the base. What is the area of the largest possible cross section?

3. Trapezoid $ABCD$ has vertices $A(0, 0)$, $B(6, 0)$, $C(3, 3)$, and $D(0, 3)$. The trapezoid is rotated around the y-axis. What is the volume of the solid formed by this rotation?

4. An empty cube-shaped container has side length 10 feet. The container is filled water at a rate of 12 cubic feet per minute while water leaks from the bottom of the container at a rate of 3 cubic feet per minute. How long will it take the container to overflow? Round to the nearest minute.

☐ **LESSON 170**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. A right triangle is formed by three lines $x = 1$, $y = 4$, and $y = x - 1$. What are the coordinates of the circumcenter of the triangle?

2. $\triangle PQR$ has vertices $P(2, 3)$, $Q(2, -3)$, and $R(-4, -3)$. What are the coordinates of the centroid of the triangle?

3. A circle circumscribes a square whose vertices are at (0, 0), (4, 0), (4, 4), and (0, 4). What is the standard equation of the circle? (*Hint*: The diagonals of the square are diameters of the circumcircle.)

☐ **LESSON 172**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. Emma, Brian, and 6 of their friends are seated randomly in a row of 8 seats. What is the probability that Emma and Brian sit next to each other?

2. The letters W, A, T, E, and R are rearranged in a random order. What is the probability that no two consonants come together?

☐ **LESSON 174**

Solve if you have not done the problem(s) marked "HONORS" in the online worksheets.

1. A rhombus is formed by connecting the midpoints of the sides of a rectangle with width 10 cm and height 6 cm. A point is randomly selected in the rectangle. What is the probability that the point is in the rhombus?

Answers

LESSON 8

1. false; Angles in a linear pair are supplementary.
2. true
3. true
4. false; Two obtuse angles add up to more than 180°, so they cannot be supplementary.
5. Complementary angles add up to 90°. So, if they are congruent, then each must measure 45°.
6. The two angles in a linear pair add up to 180°. So, if they are congruent, then each must measure 90°.
7. Vertical angles are congruent. So, if they are complementary, then each must measure 45°.
8. A right angle measures 90°, so the supplement of a right angle measures 180 − 90 = 90° as well.

LESSON 9

1. Any two adjacent angles of a parallelogram are consecutive interior angles, so they are supplementary.
2. $\angle 1$ and $\angle 2$ are supplementary and add up to 180°.
   $\angle 3$ and $\angle 4$ are supplementary and add up to 180°.
   So, the four angles add up to 360°.

LESSON 10

1. $l \parallel m$ because corresponding angles are congruent when lines l and m are cut by transversal u.
   $u \parallel v$ because alternate exterior angles are congruent when lines u and v are cut by transversal l.
2. $\overline{AF} \parallel \overline{BE}$ because $\angle AFE \cong \angle BED$ as corresponding angles when $\overline{AF}$ and $\overline{BE}$ are cut by transversal $\overline{FD}$.
   $\overline{AC} \parallel \overline{FD}$ because $\angle ABE \cong \angle BED$ as alternate interior angles when $\overline{AC}$ and $\overline{FD}$ are cut by transversal $\overline{BE}$

LESSON 11

1. Use the interior angle sum of $\triangle ABC$.
   $m\angle ECF = 180 - 90 - m\angle A = 180 - 90 - 62 = 28°$
2. Use the interior angle sum of $\triangle BCD$.
   $m\angle EBF = 180 - 90 - m\angle D = 180 - 90 - 70 = 20°$
3. $m\angle ABE$ and $m\angle EBF$ are complementary.
   $m\angle ABE = 90 - m\angle EBF = 90 - 20 = 70°$
4. $m\angle AED$ is an exterior angle of $\triangle ABE$.
   $m\angle AED = m\angle A + m\angle ABE = 62 + 70 = 132°$

LESSON 13

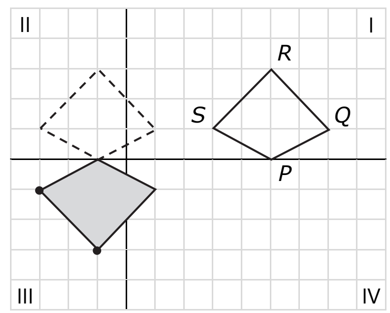
1. $180(n - 2) = 150n$
   $30n = 360$
   $n = 12$ sides
2. $360 = 72n$
   $n = 5$ sides

LESSON 16

1. The translation maps (x, y) to $(x, y - 2)$.
   $(x, y - 2)$ becomes (0, 0) when $x = 0$ and $y = 2$.
   So, this translation maps (0, 2) to the origin.
2. The translation maps (x, y) to $(x - 4, y)$.
   $(x - 4, y)$ becomes (0, 0) when $x = 4$ and $y = 0$.
   So, this translation maps (4, 0) to the origin.
3. The translation maps (x, y) to $(x - 2, y + 5)$.
   $(x - 2, y + 5)$ becomes (0, 0) when $x = 2$ and $y = -5$.
   So, this translation maps (2, −5) to the origin.

LESSON 17

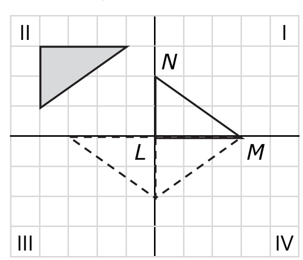
1. Two vertices are in Quadrant III.



2. The translation rule is $(x, y) \to (x - 7, y + 5)$. So, the new coordinates of B are (−10, 6).
3. The point before the reflection is (−1, 6). This means that the translation maps (2, 5) to (−1, 6). So, the translation rule is $(x, y) \to (x - 3, y + 1)$.
   Apply these transformations to Q. The translation maps $Q(4, -3)$ to $Q'(1, -2)$. The reflection maps $Q'(1, -2)$ to $Q''(-2, 1)$. So, the new coordinates of Q are (−2, 1).

LESSON 18

1. The point before the rotation is (−2, −3). The point before the reflection is (−3, −2). This means that the translation maps (2, 5) to (−3, −2). So, the translation rule is $(x, y) \to (x - 5, y - 7)$.
   Apply these transformations to Q. The translation maps $Q(9, 7)$ to $Q'(4, 0)$. The reflection maps $Q'(4, 0)$ to $Q''(0, 4)$. The rotation maps $Q''(0, 4)$ to $Q'''(0, -4)$. So, the new coordinates of Q are (0, −4).
2. The image is in Quadrant II.



LESSON 21

1. a rotation of 180° about the origin
2. The translation maps (x, y) to $(x, y-3)$.
   The reflection maps $(x, y-3)$ to $(x, -(y-3))$.
   So, the rule is $(x, y) \rightarrow (x, -y+3)$.
3. The reflection maps (x, y) to (y, x).
   The dilation maps (y, x) to $(2y, 2x)$.
   So, the rule is $(x, y) \rightarrow (2y, 2x)$.
4. A) false; A reflection creates a mirror image.
   B) false; A translation moves every point of the figure the same distance in the same direction.
   C) true
   D) true; A rotation is a rigid transformation.
   E) false; A 90° clockwise rotation is the same as a 270° counterclockwise rotation.
   F) true
   G) false; A dilation with a scale factor of 3 makes a figure three times bigger.
   H) false; A composition of rigid transformations is also a rigid transformation.
   I) true
   J) false; A composition of reflections over two intersecting lines can be described as a single rotation.

LESSON 23

1. The sum of three consecutive integers is three times the second integer.
2. The sum of the first n positive odd integers is n^2.

LESSON 29

1. Statements (Reasons)
   1. $\angle 1 \cong \angle 2$ (Given)
   2. $\angle 1 \cong \angle 3$ (Vertical angles are congruent.)
   3. $\angle 2 \cong \angle 3$ (Transitive Property)
   4. $\angle 2 \cong \angle 4$ (Vertical angles are congruent.)
   5. $\angle 3 \cong \angle 4$ (Transitive Property)
2. Statements (Reasons)
   1. $\overrightarrow{PE} \perp \overrightarrow{PG}, \overrightarrow{PF} \perp \overrightarrow{PH}$ (Given)
   2. $\angle EPG$ and $\angle FPH$ are right angles. (Definition of perpendicular)
   3. $\angle 1$ is complementary to $\angle 2$, $\angle 3$ is complementary to $\angle 2$ (Definition of complementary angles)
   4. $\angle 1 \cong \angle 3$ (Angles complementary to the same angle are congruent. See Theorem 29.2.)
3. Statements (Reasons)
   1. $\angle 1$ is complementary to $\angle 3$, $\angle 2$ is complementary to $\angle 4$, $\angle 3 \cong \angle 4$ (Given)
   2. $m\angle 1 + m\angle 3 = 90°$, $m\angle 2 + m\angle 4 = 90°$ (Definition of complementary angles)
   3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$ (Transitive Property)
   4. $m\angle 3 = m\angle 4$ (Definition of congruent angles)
   5. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$ (Substitution Property)
   6. $m\angle 1 = m\angle 2$ (Subtraction Property)
   7. $\angle 1 \cong \angle 2$ (Definition of congruent angles)
4. Statements (Reasons)
   1. $\angle 1$ is supplementary to $\angle 3$, $\angle 2$ is supplementary to $\angle 4$, $\angle 3 \cong \angle 4$ (Given)
   2. $m\angle 1 + m\angle 3 = 180°$, $m\angle 2 + m\angle 4 = 180°$ (Definition of supplementary angles)
   3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$ (Transitive Property)
   4. $m\angle 3 = m\angle 4$ (Definition of congruent angles)
   5. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$ (Substitution Property)
   6. $m\angle 1 = m\angle 2$ (Subtraction Property)
   7. $\angle 1 \cong \angle 2$ (Definition of congruent angles)

LESSON 30

1. Statements (Reasons)
   1. $s \parallel t,\ u \parallel v$ (Given)
   2. $\angle 1 \cong \angle 5$, $\angle 5 \cong \angle 13$ (If lines are parallel, then corresponding angles are congruent.)
   3. $\angle 1 \cong \angle 13$ (Transitive Property)
2. Statements (Reasons)
   1. $s \parallel t,\ u \parallel v$ (Given)
   2. $\angle 6 \cong \angle 2$ (If lines are parallel, then corresponding angles are congruent.)
   3. $\angle 2 \cong \angle 11$ (If lines are parallel, then alternate interior angles are congruent.)
   4. $\angle 6 \cong \angle 11$ (Transitive Property)
3. Statements (Reasons)
   1. $s \parallel t$, $\angle 3 \cong \angle 14$ (Given)
   2. $\angle 3 \cong \angle 7$ (If lines are parallel, then corresponding angles are congruent.)
   3. $\angle 7 \cong \angle 14$ (Transitive Property)
   4. $u \parallel v$ (If alternate exterior angles are congruent, then lines are parallel.)

LESSON 31

1. Statements (Reasons)

1. $a \parallel b$, $b \perp d$, $c \perp d$ (Given)
2. $b \parallel c$ (Lines that are perpendicular to the same line are parallel. See Theorem 31.2.)
3. $a \parallel c$ (Transitive Property of Parallel Lines)

2. Statements (Reasons)

1. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ (Given)
2. $a \parallel b$ (If alternate exterior angles are congruent, then lines are parallel.)
3. $b \parallel c$ (If alternate interior angles are congruent, then lines are parallel.)
4. $a \parallel c$ (Transitive Property of Parallel Lines)

LESSON 32

1.
3. Angles in a triangle add up to 180°.
4. Substitution Property
5. Subtraction Property

2. Statements (Reasons)

1. $\overline{AC} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$ (Given)
2. $\angle ACB$ are $\angle ADC$ right angles. (Definition of perpendicular)
3. $\triangle ABC$ and $\triangle ACD$ are right triangles. (Definition of right triangles)
4. In $\triangle ABC$, $\angle A$ and $\angle B$ are complementary. In $\triangle ACD$, $\angle A$ and $\angle ACD$ are complementary. (Acute angles in a right triangle are complementary. See Problem 1.)
5. $\angle B \cong \angle ACD$ (Angles complementary to the same angle are congruent. See Theorem 29.2.)

LESSON 33

1. Rotate $\overline{MP}$ and line l by 180° about M. Then $\overline{MP}$ maps onto $\overline{MQ}$ because

- Ray $\overrightarrow{MP}$ maps onto ray $\overrightarrow{MQ}$.
- P and Q are equidistant from M.

Also, line l maps onto line m because

- P is on line l and P maps onto Q, so line l maps onto a line through Q.
- 180° rotations map a line onto a parallel line if the center of rotation is not on the line, so line l maps onto a line parallel to line l.
- Line m is the only line through Q parallel to l.

This means that our rotation maps $\angle 1$ formed by $\overline{MP}$ and line l onto $\angle 2$ formed by $\overline{MQ}$ and line m. Rotations are rigid transformations, so $\angle 1 \cong \angle 2$.

LESSON 36

1. Statements (Reasons)

1. $\overline{BD} \perp \overline{CE}$ (Given)
2. $\angle BCE$ and $\angle DCE$ are right angles. (Def. of perpendicular)
3. $\angle BCE \cong \angle DCE$ (All right angles are congruent.)
4. C bisects $\overline{BD}$. (Given)
5. $\overline{BC} \cong \overline{DC}$ (Def. of bisect)
6. $\overline{CE} \cong \overline{CE}$ (Reflexive Property)
7. $\triangle BCE \cong \triangle DCE$ (SAS, Steps 3, 5, and 6)

2. Statements (Reasons)

1. $ABCD$ is a kite. (Given)
2. $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$ (Def. of kite)
3. $\overline{AC} \cong \overline{AC}$ (Reflexive Property)
4. $\triangle ABC \cong \triangle ADC$ (SSS)

LESSON 37

1. Statements (Reasons)

1. $\overline{FG} \parallel \overline{EH}$, $\overline{FE} \parallel \overline{GH}$ (Given)
2. $\angle FEG \cong \angle HGE$, $\angle FGE \cong \angle HEG$ (If lines are ∥, then alternate interior angles are ≅.)
3. $\overline{GE} \cong \overline{EG}$ (Reflexive Property)
4. $\triangle EFG \cong \triangle GHE$ (ASA)

2. Statements (Reasons)

1. $\overline{AB} \cong \overline{DE}$ (Given)
2. $\overline{AB} \perp \overline{BE}$, $\overline{DE} \perp \overline{BE}$ (Given)
3. $\angle ABC$ and $\angle DEC$ are right angles. (Def. of perpendicular)
4. $\angle ABC \cong \angle DEC$ (All right angles are congruent.)
5. $\angle ACB \cong \angle DCE$ (Vertical angles are congruent.)
6. $\triangle ABC \cong \triangle DEC$ (AAS, Steps 1, 4, and 5)

LESSON 38

1. Statements (Reasons)

1. $\overline{AB} \parallel \overline{DC}$, $\overline{AC} \parallel \overline{DE}$ (Given)
2. $\angle B \cong \angle DCE$, $\angle ACB \cong \angle E$ (If lines are ∥, then corresponding angles are ≅.)
3. C bisects $\overline{BE}$. (Given)
4. $\overline{BC} \cong \overline{CE}$ (Def. of bisect)
5. $\triangle ABC \cong \triangle DCE$ (ASA, Steps 2 and 4)
6. $\overline{AC} \cong \overline{DE}$ (CPCTC)

2. Statements (Reasons)
   1. $DEFG$ is a square. (Given)
   2. $\overline{DE} \cong \overline{EF} \cong \overline{FG} \cong \overline{GD}$ (Def. of square)
   3. $\overline{DF} \cong \overline{FD}$ (Reflexive Property)
   4. $\triangle DEF \cong \triangle DGF$ (SSS)
   5. $\angle EDF \cong \angle GDF$ (CPCTC)

LESSON 39

1. 3. All right angles are congruent.
   5. Def. of bisect (or segment bisector)
   6. Reflexive Property
   7. SAS
   10. Reflexive Property
   11. SAS
   12. CPCTC

2. Statements (Reasons)
   1. $\overline{AB} \cong \overline{AD}$, $\overline{BE} \cong \overline{DE}$ (Given)
   2. $\overline{AE} \cong \overline{AE}$ (Reflexive Property)
   3. $\triangle ABE \cong \triangle ADE$ (SSS)
   4. $\angle BAE \cong \angle DAE$ (CPCTC)
   5. $\overline{AC} \cong \overline{AC}$ (Reflexive Property)
   6. $\triangle ABC \cong \triangle ADC$ (SAS, Steps 1, 4, and 5)

LESSON 40

1. $90 + 2a = 180$
$a = 45$
$36 + 2(a + b) = 180$
$b = 27$

2. $a = 72$
$b = 180 - a = 108$
$b + 2c = 180$
$c = 36$

3. $a = 180 - 62 = 118$
$a + 2b = 180$
$b = 31$
$62 + 2c = 180$
$c = 59$

LESSON 42

1. 2. Def. of congruent segments
   4. Subtraction Property
   6. Transitive Property (Steps 4 and 5)
   8. Def. of congruent segments
   9. SSS

LESSON 44

1. translation

2. The intercepts are at (−1, 0) and (0, 1). These are translated to (−1, 3) and (0, 4), then reflected to (1, 3) and (0, 4). The slope-intercept equation of a line passing through (1, 3) and (0, 4) is $y = -x + 4$.

LESSON 46

1. *Answer formats may vary.*
Statements (Reasons)
   1. a. $\overline{BQ} \cong \overline{QC}$ (Given)
      b. $\overline{AB} \parallel \overline{CR}$ (Construction)
      c. $\angle B \cong \angle QCR$ (Alternate interior ∠s are ≅.)
      d. $\angle BQP \cong \angle CQR$ (Vertical ∠s are ≅.)
      e. $\triangle BQP \cong \triangle CQR$ (ASA, Steps 1a, 1c and 1d)
   2. a. $\overline{QP} \cong \overline{QR}$ (CPCTC, Step 1e)
      b. $\overline{PB} \cong \overline{RC}$ (CPCTC, Step 1e)
   3. a. $\overline{BP} \cong \overline{PA}$ (Given)
      b. $\overline{PA} \cong \overline{RC}$ (Transitive Property, Steps 2b and 3a)
   4. a. $\angle APC \cong \angle RCP$ (Alternate interior ∠s are ≅.)
      b. $\overline{PC} \cong \overline{CP}$ (Reflexive Property)
      c. $\triangle APC \cong \triangle RCP$ (SAS, Steps 3b, 4a, and 4b)
   5. a. $\angle PCA \cong \angle CPR$ (CPCTC, Step 4c)
   6. a. $\overline{PQ} \parallel \overline{AC}$ (Alternate interior ∠s are ≅.)
   7. a. $\overline{PR} \cong \overline{AC}$ (CPCTC, Step 4c)
   8. a. $PR = PQ + QR$ (Segment Addition Postulate)
      b $PQ = QR$ (Def. of congruent segments, Step 2a)
      c. $PR = PQ + PQ$ (Substitution Property)
      d. $PR = 2PQ$ (Simplify.)
      e. $PR = AC$ (Def. of congruent segments, Step 7a)
      f. $AC = 2PQ$ (Transitive Property)

LESSON 47

1. Statements (Reasons)
   1. $\overline{CS}$, $\overline{CT}$, and $\overline{CU}$ are perpendicular bisectors. (Given)
   2. $CX = CY$, $CY = CZ$, $CZ = CX$ (Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.)
   3. $CX = CY = CZ$ (Transitive Property)

LESSON 48

1. Statements (Reasons)
   1. $\overline{CX}$, $\overline{CY}$, and $\overline{CZ}$ are angle bisectors.
      $\overline{XY} \perp \overline{CS}$, $\overline{YZ} \perp \overline{CT}$, $\overline{XZ} \perp \overline{CU}$ (Given)
   2. $CS = CU$, $CS = CT$, $CT = CU$ (Any point on the bisector of an angle is equidistant from the sides of the angle.)
   3. $CS = CT = CU$ (Transitive Property)

LESSON 49

1 ~ 3. *Answer formats may vary.*

1. Statements (Reasons)

1. $\overline{PQ} \cong \overline{PR}$, $\overline{PK}$ is a median. (Given)
3. K is a midpoint. (Def. of median)
4. $\overline{QK} \cong \overline{RK}$ (Def. of midpoint)
5. $\overline{PK} \cong \overline{PK}$ (Reflexive Property)
6. $\triangle PQK \cong \triangle PRK$ (SSS)
7. $\angle QPK \cong \angle RPK$ (CPCTC)
8. $\overline{PK}$ is an angle bisector. (Def. of angle bisector)

2. Statements (Reasons)

1. $\overline{PQ} \cong \overline{PR}$, $\overline{PK}$ is an angle bisector. (Given)
2. $\angle QPK \cong \angle RPK$ (Def. of angle bisector)
3. $\overline{PK} \cong \overline{PK}$ (Reflexive Property)
4. $\triangle PQK \cong \triangle PRK$ (SAS)
5. $\overline{QK} \cong \overline{RK}$ (CPCTC)
6. K is a midpoint. (Def. of midpoint)
7. $\overline{PK}$ is a median. (Def. of median)

3. Statements (Reasons)

1. a. $\overline{XE}$ and $\overline{ZD}$ are medians. (Given)
   b. D and E are midpoints. (Def. of medians)
   c. $\overline{DE}$ is a midsegment of $\triangle XYZ$. (Def. of midsegment)
   d. $\overline{DE} \parallel \overline{XZ}$ (Triangle Midsegment Theorem [46.1])
   e. G and F are midpoints. (Construction)
   f. $\overline{GF}$ is a midsegment of $\triangle CXZ$. (Def. of midsegment)
   g. $\overline{GF} \parallel \overline{XZ}$ (Triangle Midsegment Theorem [46.1])
   h. $\overline{DE} \parallel \overline{GF} \parallel \overline{XZ}$ (Transitive Property, Steps 1d and 1g)
2. a. $DE = XY/2$, $GF = XY/2$ (Triangle Midsegment Theorem [46.1])
   b. $DE = GF$ (Transitive Property)
   c. $\overline{DE} \cong \overline{GF}$ (Def. of congruent segments)
3. a. $\angle CDE \cong \angle CFG$, $\angle CED \cong \angle CGF$ (If lines parallel, then alternate interior ∠s are ≅.)
   b. $\triangle CDE \cong \triangle CFG$ (ASA, Steps 2c and 3a)
4. a. $\overline{CE} \cong \overline{CG}$ (CPCTC, Step 3b)
   b. $\overline{CG} \cong \overline{GX}$ (Def. of midpoint, Step 1e)
   c. $\overline{CE} \cong \overline{CG} \cong \overline{GX}$ (Transitive Property)
   d. $CE = CG = GX$ (Def. of congruent segments)

(The proof continues on the next column.)

3.
5. a. $CX = CG + GX$ (Segment Addition Postulate)
   b. $CX = CE + CE$ (Substitution Property)
   c. $CX = 2CE$ (Simplify.)
6. a. $\overline{CD} \cong \overline{CF}$ (CPCTC, Step 3b)
   b. $\overline{CF} \cong \overline{FZ}$ (Def. of midpoint, Step 1e)
   c. $\overline{CD} \cong \overline{CF} \cong \overline{FZ}$ (Transitive Property)
   d. $CD = CF = FZ$ (Def. of congruent segments)
   e. $CZ = CF + FZ$ (Segment Addition Postulate)
   f. $CZ = CD + CD$ (Substitution Property)
   g. $CZ = 2CD$ (Simplify)

LESSON 50

1 ~ 4. *Answer formats may vary.*

1. Statements (Reasons)

1. $\overline{PQ} \cong \overline{PR}$, $\overline{PK}$ is a median. (Given)
3. K is a midpoint. (Def. of median)
4. $\overline{QK} \cong \overline{RK}$ (Def. of midpoint)
5. $\overline{PK} \cong \overline{PK}$ (Reflexive Property)
6. $\triangle PQK \cong \triangle PRK$ (SSS)
7. $\angle PKQ \cong \angle PKR$ (CPCTC)
8. $\angle PKQ$ & $\angle PKR$ are a linear pair. (Def. of linear pair)
8. $\overline{PK} \perp \overline{QR}$ (Two lines intersecting to form a linear pair of congruent angles are perpendicular. See Theorem 31.3.)
9. $\overline{PK}$ is an altitude. (Def. of altitude)

2. Statements (Reasons)

1. $\overline{PQ} \cong \overline{PR}$, $\overline{PK}$ is an altitude. (Given)
2. $\overline{PK} \perp \overline{QR}$ (Def. of altitude)
3. $\angle PKQ$ and $\angle PKR$ are right angles. (Def. of perpendicular)
4. $\triangle PQK$ and $\triangle PRK$ are right triangles. (Def. of right triangle)
5. $\overline{PK} \cong \overline{PK}$ (Reflexive Property)
6. $\triangle PQK \cong \triangle PRK$ (HL, Steps 1 and 5)
7. $\overline{QK} \cong \overline{RK}$ (CPCTC)
8. K is a midpoint. (Def. of midpoint)
9. $\overline{PK}$ is a median. (Def. of median)

3. Statements (Reasons)

1. $\overline{PE}$ is an altitude of isosceles $\triangle PQF$. (Given)
2. $\overline{PE}$ is a median of $\triangle PQF$. (An altitude of an isosceles triangle is also a median. See Problem 2.)
3. E is the midpoint of $\overline{QF}$. (Def. of median)
4. $\overline{QE} \cong \overline{EF}$ (Def. of midpoint)
5. $QE = EF$ (Def. of congruent segments)
6. $QF = QE + EF = 2QE$ (Segment Addition Postulate, Substitution Property)
7. $\overline{PF}$ is a median of $\triangle PQR$. (Given)
8. F is the midpoint of $\overline{QR}$. (Def. of median)
9. $\overline{QF} \cong \overline{FR}$ (Def. of midpoint)
10. $QF = FR$ (Def. of congruent segments)
11. $QR = QF + FR = 2QF = 2(2QE) = 4QE$ (Segment Addition Postulate, Substitution Property)

4. Statements (Reasons)

1. $\triangle PQF$ is isosceles with $\overline{PQ} \cong \overline{PF}$. (Given)
2. $\angle Q \cong \angle PFE$ (Base Angles Theorem [40.1])
3. $m\angle Q = m\angle PFE$ (Def. of congruent angles)
4. $\triangle FPR$ is isosceles with $\overline{FP} \cong \overline{FR}$. (Given)
5. $\angle R \cong \angle FPR$ (Base Angles Theorem [40.1])
6. $m\angle R = m\angle FPR$ (Def. of congruent angles)
7. $m\angle PFE = m\angle R + m\angle FPR$ (Triangle Exterior Angle Theorem [32.2])
8. $m\angle Q = m\angle R + m\angle R = 2m\angle R$ (Substitution Property, Steps 3, 6 and 7)

5. altitude, median, perpendicular bisector, angle bisector

LESSON 51

1. *Answer formats may vary.*

A midsegment is parallel to the third side (Theorem 46.1). $\overline{DE}$ is a midsegment of $\triangle ABC$, so $\overline{DE} \parallel \overline{AC}$.

A line perpendicular to one of two parallel lines is also perpendicular to the other (Theorem 31.1). $\overline{DE} \parallel \overline{AC}$ and $\overline{AC} \perp \overline{BC}$, so $\overline{DE} \perp \overline{BC}$.

For the same reason, $\overline{DF} \parallel \overline{BC}$ and thus $\overline{DF} \perp \overline{AC}$.

2. median (midpoint F of $\overline{AC}$),
altitude ($\overline{DF} \perp \overline{AC}$),
perpendicular bisector (both altitude and median),
angle bisector ($\triangle ADF \cong \triangle CDF$ by SAS)

Notice that $\triangle ADF \cong \triangle CDF$ by SAS because $\overline{AF} \cong \overline{CF}$ by the definition of midpoint, $\angle DFA \cong \angle DFC$ as right angles, and $\overline{DF} \cong \overline{DF}$ by the Reflexive Property.

3. Because $\overline{DE} \perp \overline{BC}$ and E bisects $\overline{BC}$, $\overline{DE}$ is the perpendicular bisector of $\overline{BC}$. Similarly, $\overline{DF}$ is the perpendicular bisector of $\overline{AC}$.

Because D is where perpendicular bisectors of $\triangle ABC$ intersect, it is the circumcenter of $\triangle ABC$ by definition.

The circumcenter of a right triangle is the midpoint of the hypotenuse.

LESSON 52

1 ~ 3. *Answer formats may vary.*

1. Statements (Reasons)

1. a. $m\angle B = m\angle 1 + m\angle 3$ (Angle Addition Postulate)
   b. $m\angle B > m\angle 1$ (Def. of greater than)
2. a. $m\angle 2 = m\angle A + m\angle 3$ (Triangle Exterior Angle Theorem [32.2])
   b. $m\angle 2 > m\angle A$ (Def. of greater than)
3. a. $BC = DC$ (Construction)
   b. $m\angle 1 = m\angle 2$ (Base Angles Theorem [40.1].)
4. a. $m\angle B > m\angle 2$ (Substitution, Steps 1b and 3b)
   b. $m\angle B > m\angle A$ (Transitive Prop., Steps 2b and 4a)

2.

1. $m\angle B < m\angle A$ (Triangle Side-Angle Theorem [52.1])
2. $m\angle B = m\angle A$ (Base Angles Theorem [40.1])
3. Both cases contradict the given statement. So, our assumption is false and $AC > BC$ must be true.

3. Statements (Reasons)

1. a. $m\angle ACD = m\angle 2 + m\angle 3$ (Angle Addition Post.)
   b. $m\angle ACD > m\angle 2$ (Def. of greater than)
2. a. $BC = BD$ (Construction)
   b. $m\angle 1 = m\angle 2$ (Base Angles Theorem [40.1])
   c. $m\angle ACD > m\angle 1$ (Substitution, Steps 1b and 2b)
3. a. $AD > AC$ (Apply Theorem 52.1 to $\triangle ADC$.)
4. a. $AD = AB + BD$ (Segment Addition Postulate)
   b. $AD = AB + BC$ (Substitution, Steps 2a and 4a)
   c. $AB + BC > AC$ (Substitution, Steps 3a and 4b)

LESSON 53

1 ~ 2. *Answer formats may vary.*

1.

1. Base angles of an isosceles triangle are congruent. $\triangle ABD$ is isosceles, so $m\angle 3 = m\angle 1 + m\angle 2$.
2. Angles must be positive. $m\angle 3 = m\angle 1 + m\angle 2$ and $m\angle 1 > 0$, so $m\angle 3 > m\angle 2$ by the definition of greater than.
3. $m\angle 3 + m\angle 4 > m\angle 3$ by the definition of greater than. Because $m\angle 3 > m\angle 2$, $m\angle 3 + m\angle 4 > m\angle 2$ by the Transitive Property.
4. The larger angle has the longer opposite side. In $\triangle ADF$, $m\angle 3 + m\angle 4 > m\angle 2$ and thus $AC > DF$.

2. 1. By the Hinge Theorem [53.1], $AC < DF$.
2. $\triangle ABC \cong \triangle DEF$ by SAS, so $AC = DF$.
3. Both cases contradict the given statement. So, our assumption is false and $m\angle B > m\angle E$ must be true.

LESSON 54

1. 2. Alternate interior ∠s on parallel lines are ≅.
3. Reflexive Property
4. AAS
6. The sum of two sides of a triangle is greater than the third side. See Theorem 53.3.

2. 2. Def. of bisect
3. Triangle Exterior Angle Theorem [32.2]
4. Substitution Property
6. The larger angle has the longer opposite side. See Theorem 52.2.

3. Statements (Reasons)
1. $m\angle C = 90°$ (Given)
2. $m\angle A + m\angle B + m\angle C = 180°$ (Angles in a triangle add up to 180°. See the Triangle Sum Theorem [32.1].)
3. $m\angle A + m\angle B = 180 - m\angle C$ (Subtraction Property)
4. $m\angle A + m\angle B = 180 - 90$ (Substitution Property)
5. $m\angle A + m\angle B = 90$ (Simplify.)
6. $m\angle A < 90$, $m\angle B < 90$ (Def. of less than)
7. $m\angle A < m\angle C$, $m\angle B < m\angle C$ (Substitution Property)
8. $BC < AB$, $AC < AB$ (The larger angle has the longer opposite side. See Theorem 52.2.)

4. Statements (Reasons)
1. ∠3 is an exterior angle. (Given)
2. $m\angle 3 = m\angle 1 + m\angle 2$ (Triangle Exterior Angle Theorem [32.2])
3. $m\angle 3 > m\angle 1$, $m\angle 3 > m\angle 2$ (Def. of greater than)

LESSON 56

1. Statements (Reasons)
1. ▱$LMNR$, ▱$LPQR$ (Given)
2. $\overline{MN} \cong \overline{LR}$, $\overline{LR} \cong \overline{PQ}$ (Opposite sides of a parallelogram are congruent.)
3. $\overline{MN} \cong \overline{PQ}$ (Transitive Property)

2. Statements (Reasons)
1. ▱$EFGH$, ▱$EPQR$ (Given)
2. $\angle E \cong \angle G$, $\angle E \cong \angle Q$ (Opposite angles of a parallelogram are congruent.)
3. $\angle G \cong \angle Q$ (Transitive Property)

LESSON 58

1. Statements (Reasons)
1. $ABCD$ is a rhombus. (Given)
2. $ABCD$ is a parallelogram, $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ (Def. of rhombus)
3. $\overline{BP} \cong \overline{DP}$ (Diagonals of a ▱ bisect each other.)
4. $\overline{AP} \cong \overline{AP}$ (Reflexive Property)
5. $\triangle APB \cong \triangle APD$ (SSS)
6. $\angle APB \cong \angle APD$ (CPCTC)
7. $\angle APB$ & $\angle APD$ are a linear pair. (Def. of linear pair)
8. $\overline{AC} \perp \overline{BD}$ (Two lines intersecting to form a linear pair of congruent angles are perpendicular. See Theorem 31.3.)

2. Statements (Reasons)
1. $ABCD$ is a parallelogram, $\overline{AC} \perp \overline{BD}$ (Given)
2. $\angle APB$ and $\angle APD$ are right angles. (Def. of ⊥)
3. $\angle APB \cong \angle APD$ (All right angles are congruent.)
4. $\overline{BP} \cong \overline{DP}$ (Diagonals of a ▱ bisect each other.)
5. $\overline{AP} \cong \overline{AP}$ (Reflexive Property)
6. $\triangle APB \cong \triangle APD$ (SAS)
7. $\overline{AB} \cong \overline{AD}$ (CPCTC)
8. $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$ (Opposite sides of a ▱ are congruent.)
9. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ (Transitive Property)
10. $ABCD$ is a rhombus. (Def. of rhombus)

3. Statements (Reasons)
1. $ABCD$ is a rhombus. (Given)
2. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ (Def. of rhombus)
3. $\overline{AC} \cong \overline{AC}$ (Reflexive Property)
4. $\triangle ABC \cong \triangle ADC$ (SSS)
5. $\angle BAC \cong \angle DAC$, $\angle BCA \cong \angle DCA$ (CPCTC)
6. $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$. (Def. of bisect)
7. Similarly, $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

4. Statements (Reasons)
1. $ABCD$ is a ▱, $\overline{AC}$ bisects $\angle BAD$ & $\angle BCD$. (Given)
2. $\angle BAC \cong \angle DAC$, $\angle BCA \cong \angle DCA$ (Def. of bisect)
3. $\overline{AC} \cong \overline{AC}$ (Reflexive Property)
4. $\triangle ABC \cong \triangle ADC$ (ASA)
5. $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$ (CPCTC)
6. $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$ (Opposite sides of a ▱ are congruent.)
7. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ (Transitive Property)
8. $ABCD$ is a rhombus. (Def. of rhombus)

5. Statements (Reasons)
   1. $EFGH$ is a rectangle. (Given)
   2. $\angle EFG$ and $\angle FEH$ are right angles. (Def. of rectangle)
   3. $\angle EFG \cong \angle FEH$ (All right angles are congruent.)
   4. $\overline{FG} \cong \overline{EH}$ (Opposite sides of a ▱ are congruent.)
   5. $\overline{EF} \cong \overline{FE}$ (Reflexive Property)
   6. $\triangle EFG \cong \triangle FEH$ (SAS)
   7. $\overline{EG} \cong \overline{FH}$ (CPCTC)

6. Statements (Reasons)
   1. $EFGH$ is a parallelogram, $\overline{EG} \cong \overline{FH}$ (Given)
   2. $\overline{FG} \cong \overline{EH}$ (Opposite sides of a ▱ are congruent.)
   3. $\overline{EF} \cong \overline{FE}$ (Reflexive Property)
   4. $\triangle EFG \cong \triangle FEH$ (SSS)
   5. $\angle EFG \cong \angle FEH$ (CPCTC)
   6. $\angle EFG \cong \angle GHE$, $\angle FEH \cong \angle HGF$ (Opposite angles of a ▱ are congruent.)
   7. $\angle EFG \cong \angle GHE \cong \angle FEH \cong \angle HGF$ (Transitive Property)
   8. $EFGH$ is a rectangle. (Def. of rectangle)

LESSON 59

1. Statements (Reasons)
   1. trapezoid $ABCD$, $\overline{BA} \cong \overline{CD}$ (Given)
   2. $\overline{BC} \parallel \overline{AD}$ (Def. of trapezoid)
   3. Draw $\overline{CK}$ such that $\overline{BA} \parallel \overline{CK}$. (Construction)
   4. $ABCK$ is a parallelogram. (Def. of parallelogram)
   5. $\overline{BA} \cong \overline{CK}$ (Opposite sides of a ▱ are congruent.)
   6. $\overline{CK} \cong \overline{CD}$ (Transitive Property, Steps 1 and 5)
   7. $\angle CKD \cong \angle D$ (Base Angles Theorem [40.1].)
   8. $\angle A \cong \angle CKD$ (Corresponding $\angle$s on $\parallel$ lines are $\cong$.)
   9. $\angle A \cong \angle D$ (Transitive Property)
   10. $\angle B$ and $\angle A$ are supplementary; $\angle C$ and $\angle D$ are supplementary. (Consecutive interior $\angle$s on $\parallel$ lines are supplementary.)
   11. $\angle B \cong \angle C$ (Angles supplementary to congruent angles are congruent. See Theorem 29.3.)

2. Statements (Reasons)
   1. trapezoid $ABCD$, $\angle A \cong \angle D$ (Given)
   2. $\overline{BC} \parallel \overline{AD}$ (Def. of trapezoid)
   3. Draw $\overline{CK}$ such that $\overline{BA} \parallel \overline{CK}$. (Construction)
   4. $\angle A \cong \angle CKD$ (Corresponding $\angle$s on $\parallel$ lines are $\cong$.)
   5. $\angle CKD \cong \angle D$ (Transitive Property, Steps 1 and 4)
   6. $\overline{CK} \cong \overline{CD}$ (Base Angles Converse [40.2])
   7. $ABCK$ is a parallelogram. (Def. of parallelogram)
   8. $\overline{BA} \cong \overline{CK}$ (Opposite sides of a ▱ are congruent.)
   9. $\overline{BA} \cong \overline{CD}$ (Transitive Property, Steps 6 and 8)

3. Statements (Reasons)
   1. trapezoid $ABCD$, $\overline{BA} \cong \overline{CD}$ (Given)
   2. $\angle BAD \cong \angle CDA$ (Base angles of an isosceles trapezoid are congruent. See Theorem 59.2.)
   3. $\overline{AD} \cong \overline{AD}$ (Reflexive Property)
   4. $\triangle BAD \cong \triangle CDA$ (SAS)
   5. $\overline{BD} \cong \overline{CA}$ (CPCTC)

4. Statements (Reasons)
   1. trapezoid $ABCD$, $\overline{BD} \cong \overline{CA}$ (Given)
   2. $\overline{BC} \parallel \overline{AD}$ (Def. of trapezoid)
   3. Draw $\overline{CK}$ such that $\overline{CK} \parallel \overline{BD}$ and K is on $\overleftrightarrow{AD}$. (Construction)
   4. $BCKD$ is a parallelogram. (Def. of parallelogram)
   5. $\overline{BD} \cong \overline{CK}$ (Opposite sides of a ▱ are congruent.)
   6. $\overline{CA} \cong \overline{CK}$ (Transitive Property, Steps 1 and 5)
   7. $\angle CAD \cong \angle K$ (Base Angles Theorem [40.1].)
   8. $\angle K \cong \angle BDA$ (Corresponding $\angle$s on $\parallel$ lines are $\cong$.)
   9. $\angle BDA \cong \angle CAD$ (Transitive Property, Steps 7 and 8)
   10. $\overline{AD} \cong \overline{AD}$ (Reflexive Property)
   11. $\triangle BDA \cong \triangle CAD$ (SAS, Steps 1, 9, and 10)
   12. $\overline{BA} \cong \overline{CD}$ (CPCTC)

LESSON 60

1. *Answer formats may vary.*

Statements (Reasons)

1. a. trapezoid $PQRS$ with midsegment $\overline{MN}$ (Given)
   b. M and N are midpoints. (Def. of midsegment)
   c. $\overline{QN} \cong \overline{NR}$ (Def. of midpoint)
   d. $\overline{PQ} \parallel \overline{ST}$ (Def. of trapezoid)
   e. $\angle PQN \cong \angle TRN$, $\angle QPN \cong \angle RTN$ (Alternate interior ∠s on parallel lines are congruent.)
   f. $\triangle PQN \cong \triangle TRN$ (AAS)
2 a. $\overline{PN} \cong \overline{NT}$ (CPCTC, Step 1f)
   b. N is the midpoint of $\overline{PT}$. (Def. of midpoint)
   c. $\overline{MN}$ is a midsegment of $\triangle PST$. (Def. of midsegment)
3. a. $\overline{MN} \parallel \overline{ST}$ (A midsegment is parallel to the third side. See Theorem 46.1.)
   b. $\overline{MN} \parallel \overline{SR} \parallel \overline{PQ}$ (Transitive Property, Steps 1d and 3a)
4. a. $\overline{PQ} \cong \overline{RT}$ (CPCTC, Step 1f)
   b. $QP = RT$ (Def. of congruent segments)
   c. $MN = ST/2$ (A midsegment is half the length of the third side. See Theorem 46.1.)
   d. $ST = SR + RT$ (Segment Addition Postulate)
   e. $ST = SR + PQ$ (Substitution, Steps 4b & 4d)
   f. $MN = (SR + PQ)/2$ (Substitution)

2. Statements (Reasons)

1. kite $KLMN$ (Given)
2. $\overline{KL} \cong \overline{KN}$, $\overline{ML} \cong \overline{MN}$ (Def. of kite)
3. $\overline{KM} \cong \overline{KM}$ (Reflexive Property)
4. $\triangle KLM \cong \triangle KNM$ (SSS)
5. $\angle L \cong \angle N$, $\angle LKM \cong \angle NKM$, $\angle LMK \cong \angle NMK$ (CPCTC)

3. Statements (Reasons)

1. kite $KLMN$ (Given)
2. $\overline{KL} \cong \overline{KN}$, $\overline{ML} \cong \overline{MN}$ (Def. of kite)
3. $\overline{KM} \cong \overline{KM}$ (Reflexive Property)
4. $\triangle KLM \cong \triangle KNM$ (SSS)
5. $\angle LKP \cong \angle NKP$ (CPCTC)
6. $\overline{KP} \cong \overline{KP}$ (Reflexive Property)
7. $\triangle LKP \cong \triangle NKP$ (SAS, Steps 2, 5, and 6)
8. $\overline{LP} \cong \overline{NP}$, $\angle LPK \cong \angle NPK$ (CPCTC)
9. $\angle LPK$& $\angle NPK$ are a linear pair. (Def. of linear pair)
10. $\overline{KM} \perp \overline{LN}$ (Two lines intersecting to form a linear pair of congruent angles are perpendicular. See Theorem 31.3.)

LESSON 62

1. true **2.** false **3.** false **4.** true

5. false **6.** false **7.** false **8.** true

Counterexamples may vary. Sample(s):

2. a right triangle with legs 1 cm and 2 cm
a right triangle with legs 2 cm and 3 cm

3. a rectangle with width 1 cm and length 2 cm
a rectangle with width 2 cm and length 3 cm

5. a parallelogram that is a rectangle
a parallelogram that is not a rectangle

6. an equilateral triangle
an isosceles triangle with angles 50°, 65° and 65°

7. a rhombus that is a square
a rhombus that is not a square

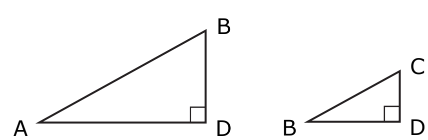
LESSON 64

1. *Answer formats may vary.*

Statements (Reasons)

1. right $\triangle ABC$ with altitude $\overline{BD}$ (Given)
2. $\angle A \cong \angle A$ (Reflexive Property)
3. $\angle ABC \cong \angle ADB$ (All right angles are congruent.)
4. $\triangle ABC \sim \triangle ADB$ (AA)
5. $\angle C \cong \angle C$ (Reflexive Property)
5. $\angle ABC \cong \angle BDC$ (All right angles are congruent.)
6. $\triangle ABC \sim \triangle BDC$ (AA)
7. $\triangle ABC \sim \triangle ADB \sim \triangle BDC$ (Transitive Property)

2. $\triangle ADB \sim \triangle BDC$



Corresponding sides are proportional.

$\frac{AD}{BD} = \frac{BD}{CD}$ → $\frac{AD}{12} = \frac{12}{6}$ → $6AD = 12(12)$, $AD = 24$

LESSON 65

1. $\triangle LMN \sim \triangle PQR$ by AA because $\angle L \cong \angle P$ and $\angle N \cong \angle R$ as corresponding angles.

$\triangle LMN \sim \triangle PXN$ by AA because $\angle L \cong \angle P$ as corresponding angles and $\angle N \cong \angle N$ by the Reflexive Property.

$\triangle PXN \sim \triangle PQR$ by AA because $\angle P \cong \angle P$ by the Reflexive Property and $\angle N \cong \angle R$ as corresponding angles.

Therefore, the three triangles are similar to each other.

2. Corresponding angles are congruent.

$m\angle M = 180 - m\angle L - m\angle XNP$

$= 180 - m\angle L - m\angle R$

$= 180 - 47 - 63 = 70°$

$m\angle Q = m\angle M = 70°$

3. Corresponding sides are proportional.

$$\frac{LM}{PQ} = \frac{LN}{PR} = \frac{LP + PN}{PN + NR}$$

$\frac{20}{25} = \frac{8 + PN}{PN + 12}$ → $20(PN + 12) = 25(8 + PN)$

$PN = 8$

LESSON 67

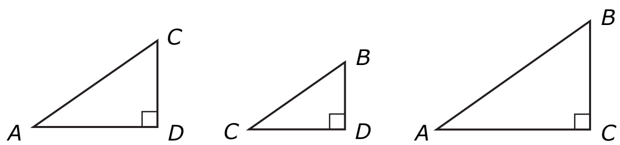
1. $\triangle ABC \sim \triangle ACD$ by AA because they both have a right angle and share $\angle A$.

$\triangle ABC \sim \triangle CBD$ by AA because they both have a right angle and share $\angle B$.

Because $\triangle ABC \sim \triangle ACD$ and $\triangle ABC \sim \triangle CBD$, their corresponding angles are all congruent. Therefore, the three triangles are similar to each other.

LESSON 68

1 ~ 2. By Theorem 67.1, these three triangles are similar.



1. Statements (Reasons)

1. right $\triangle ABC$, hypotenuse $\overline{AB}$, altitude $\overline{CD}$ (Given)
2. $\triangle ACD \sim \triangle CBD$ (Theorem 67.1)
3. $\frac{AD}{CD} = \frac{CD}{BD}$ (Corresponding sides of similar triangles are proportional.)
4. $CD^2 = AD \cdot BD$ (Cross multiply.)

2. Similarly,

$\triangle ACD \sim \triangle ABC$, so $\frac{AD}{AC} = \frac{AC}{AB}$ and $AC^2 = AD \cdot AB$.

$\triangle CBD \sim \triangle ABC$, so $\frac{BD}{BC} = \frac{BC}{BA}$ and $BC^2 = BD \cdot BA$.

LESSON 69

1. $\frac{a}{b} + 1 = \frac{c}{d} + 1$

$\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$

$\frac{a + b}{b} = \frac{c + d}{d}$

2. $\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$

$\frac{a}{b} + 1 = \frac{c}{d} + 1$

$\frac{a}{b} = \frac{c}{d}$

3. Statements (Reasons)

1. $\frac{PB}{AP} = \frac{QC}{AQ}$ (Given)
2. $\frac{PB + AP}{AP} = \frac{QC + AQ}{AQ}$ (Property of proportions)
3. $AB = AP + PB$,
   $AC = AQ + QC$ (Segment Addition Postulate)
4. $\frac{AB}{AP} = \frac{AC}{AQ}$ (Substitution Property)
5. $\angle A \cong \angle A$ (Reflexive Property)
6. $\triangle APQ \sim \triangle ABC$ (SAS, Steps 4 and 5)
7. $\angle APQ \cong \angle B$ (Corresponding angles of similar triangles are congruent.)
8. $\overline{PQ} \parallel \overline{BC}$ (If corresponding angles are congruent, then lines are parallel.)

4. Statements (Reasons)

1. $\overline{PQ} \parallel \overline{BC}$ (Given)
2. $\angle APQ \cong \angle B$ (Corresponding ∠s on ∥ lines are ≅.)
3. $\angle A \cong \angle A$ (Reflexive Property)
4. $\triangle APQ \sim \triangle ABC$ (AA)
5. $\frac{AB}{AP} = \frac{AC}{AQ}$ (Corresponding sides of similar triangles are proportional.)
6. $AB = AP + PB$,
   $AC = AQ + QC$ (Segment Addition Postulate)
7. $\frac{PB + AP}{AP} = \frac{QC + AQ}{AQ}$ (Substitution Property)
8. $\frac{PB}{AP} = \frac{QC}{AQ}$ (Property of proportions)

5. Given: $\overleftrightarrow{DL} \parallel \overleftrightarrow{EM} \parallel \overleftrightarrow{FN}$

Prove: $\frac{DE}{EF} = \frac{LM}{MN}$

Proof:

1. $\overleftrightarrow{DL} \parallel \overleftrightarrow{EM} \parallel \overleftrightarrow{FN}$ (Given)
2. Draw $\overline{DPN}$. (Construction)
3. $\frac{DE}{EF} = \frac{DP}{PN}$ in $\triangle DFN$ (Theorem 69.1)
4. $\frac{DP}{PN} = \frac{LM}{MN}$ in $\triangle DLN$ (Theorem 69.1)
5. $\frac{DE}{EF} = \frac{LM}{MN}$ (Transitive Property)

LESSON 70

1. $\frac{AP}{PX} = \frac{AQ}{QY} = \frac{AR}{RZ}$

2. $\frac{AP}{AX} = \frac{PQ}{XY} = \frac{PR}{XZ} = \frac{AR}{AZ}$

3. $\frac{XY}{YZ} = \frac{AX}{AZ}$

4. $\frac{AP}{PQ} = \frac{AR}{RQ}$

5. *Answer formats may vary.*

Statements (Reasons)

1. $\overline{DC} \parallel \overline{BP}$ (Construction)
   $\angle 1 \cong \angle 3$ (Corresponding ∠s on ∥ lines are ≅.)
2. $\angle 2 \cong \angle 4$ (Alternate interior ∠s on ∥ lines are ≅.)
3. $\angle 1 \cong \angle 2$ (Given)
   $\angle 3 \cong \angle 4$ (Transitive Property)
4. $\overline{BC} \cong \overline{PC}$ (Base Angles Converse [40.2])
   $BC = PC$ (Def. of congruent segments)
5. $\frac{AD}{DB} = \frac{AC}{PC}$ (Triangle Side Splitter Theorem [69.1])
6. $\frac{AD}{DB} = \frac{AC}{BC}$ (Substitution Property, Steps 4 and 5)

LESSON 71

1. Statements (Reasons)

1. $AB = 3AD$, $AC = 3AE$ (Given)
2. $\frac{AB}{AD} = \frac{3AD}{AD} = 3$, $\frac{AC}{AE} = \frac{3AE}{AE} = 3$ (Division property)
3. $\frac{AB}{AD} = \frac{AC}{AE}$ (Transitive Property)
4. $\angle A \cong \angle A$ (Reflexive Property)
5. $\triangle ABC \sim \triangle ADE$ (SAS)

2. Statements (Reasons)

1. $\angle B \cong \angle PCA$ (Given)
2. $\angle P \cong \angle P$ (Reflexive Property)
3. $\triangle PBC \sim \triangle PCA$ (AA)
4. $\frac{PB}{PC} = \frac{PC}{PA}$ (CSSTP, Corresponding sides of similar triangles are proportional.)
5. $PC^2 = PA \cdot PB$ (Cross multiply.)

LESSON 74

1. a. area = $(a + b)^2 = a^2 + 2ab + b^2$

b. area = 4 right triangles + small square

$= 4 \times \frac{1}{2}ab + c^2 = 2ab + c^2$

c. $a^2 + 2ab + b^2 = 2ab + c^2$ (Set the two areas equal.)

$a^2 + b^2 = c^2$ (Subtract $2ab$ from both sides.)

2. a. The bases of a trapezoid are the parallel sides, so the bases of this trapezoid are a and b.

The height of a trapezoid is the perpendicular distance between the bases, so the height of this trapezoid is $b + a$.

area = $\frac{1}{2}(a + b)(b + a) = \frac{1}{2}(a + b)^2$

b. area = three right triangles

$= \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2 = ab + \frac{1}{2}c^2$

c. $\frac{1}{2}(a + b)^2 = ab + \frac{1}{2}c^2$ (Set the two areas equal.)

$(a + b)^2 = 2ab + c^2$ (Multiply both sides by 2.)

$a^2 + 2ab + b^2 = 2ab + c^2$ (Expand the left side.)

$a^2 + b^2 = c^2$ (Subtract $2ab$ from both sides.)

LESSON 75

1. *Answer formats may vary.*

Statements (Reasons)

1. Draw a right $\triangle XYZ$ with a, b, and z. (Construction)
   $z^2 = a^2 + b^2$ (Pythagorean Theorem [74.1])
2. $c^2 = a^2 + b^2$ (Given)
   $z^2 = c^2$ (Transitive Property)
3. $z = c$ (Take the square root.)
4. $\triangle ABC \cong \triangle XYZ$ (SSS)

2. Statements (Reasons)

1. Draw a right $\triangle XYZ$ as in Problem 1. (Construction)
2. $z^2 = a^2 + b^2$ (Pythagorean Theorem [74.1])
3. $c^2 < a^2 + b^2$ (Given)
4. $c^2 < z^2$ (Substitution Property)
5. $c < z$ (Take the square root.)
6. $m\angle C < m\angle Z$ (Converse of Hinge Theorem [53.2])
7. $m\angle C < 90°$ (Substitution Property)
8. $\triangle ABC$ is acute. (Def. of acute triangle)

3. Statements (Reasons)

1. Draw a right $\triangle XYZ$ as in Problem 1. (Construction)
2. $z^2 = a^2 + b^2$ (Pythagorean Theorem [74.1])
3. $c^2 > a^2 + b^2$ (Given)
4. $c^2 > z^2$ (Substitution Property)
5. $c > z$ (Take the square root.)
6. $m\angle C > m\angle Z$ (Converse of Hinge Theorem [53.2])
7. $m\angle C > 90°$ (Substitution Property)
8. $\triangle ABC$ is obtuse. (Def. of obtuse triangle)

LESSON 76

1. Statements (Reasons)

1. $\triangle ABC$ is a 45°-45°-90° triangle. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Base Angles Converse [40.2])
3. $AB = BC$ (Def. of congruent segments)
4. $AC^2 = AB^2 + BC^2$ (Pythagorean Theorem [74.1])
5. $AC^2 = AB^2 + AB^2$ (Substitution Property)
6. $AC^2 = 2AB^2$ (Simplify.)
7. $AC = AB\sqrt{2}$ (Take the square root.)

2. Statements (Reasons)

1. $\triangle ABC$ is a 30°-60°-90° triangle. (Given)
2. Construct $\triangle ABD$ such that $BC = BD$. (Construction)

First, prove that $\triangle ABC \cong \triangle ABD$.

3. $\overline{BC} \cong \overline{BD}$ (Def. of congruent segments)
4. $\overline{AB} \cong \overline{AB}$ (Reflexive Property)
5. $\angle ABC \cong \angle ABD$ (All right angles are congruent.)
6. $\triangle ABC \cong \triangle ABD$ (SAS)

Second, prove that $\triangle ACD$ is equilateral.

7. $\angle C \cong \angle D$, $\angle CAB \cong \angle DAB$ (CPCTC)
8. $m\angle C = m\angle D$, $m\angle CAB = m\angle DAB$ (Def. of congruent angles)
9. $m\angle C$ = 60°, $m\angle CAB$ = 30° (Given)
10. $m\angle C = m\angle D$ = 60°, $m\angle CAB = m\angle DAB$ = 30° (Transitive Property)
11. $m\angle CAD = m\angle CAB + m\angle DAB$ = 30 + 30 = 60° (Angle Addition Postulate)
12. $\triangle ACD$ is equiangular. (Def. of equiangular triangle)
13. $\triangle ACD$ is equilateral. (All equiangular triangles are equilateral. See Theorem 40.4.)

Third, prove that $AC = 2BC$.

14. $AC = CD$ (Def. of equilateral triangle)
15. $CD = BC + BD$ (Segment Addition Postulate)
16. $CD = BC + BC = 2BC$ (Substitution, Steps 2 and 15)
17. $AC = 2BC$ (Transitive Property, Steps 14 and 16)

Lastly, prove that $AB = BC\sqrt{3}$.

18. $AC^2 = AB^2 + BC^2$ (Pythagorean Theorem [74.1])
19. $AB^2 = AC^2 - BC^2$ (Subtraction Property)
20. $AB^2 = (2BC)^2 - BC^2$ (Substitution, Steps 17 and 19)
21. $AB^2 = 4BC^2 - BC^2 = 3BC^2$ (Simplify.)
22. $AB = BC\sqrt{3}$ (Take the square root.)

LESSON 77

1. longer leg = $\sqrt{3}$ (shorter leg) = $6\sqrt{3}$

hypotenuse = 2 (shorter leg) = 12

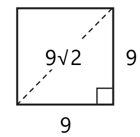
perimeter = 6 + $6\sqrt{3}$ + 12 = 18 + $6\sqrt{3}$ cm

2. An isosceles right triangle is a 45-45-90 triangle.

hypotenuse = $\sqrt{2}$ (leg) = $5\sqrt{2}$

perimeter = 5 + 5 + $5\sqrt{2}$ = 10 + $5\sqrt{2}$ in

3. A diagonal divides a square into two congruent 45-45-90 triangles. In each 45-45-90 triangle, the legs are the sides of the square, and the hypotenuse is the diagonal of the square.

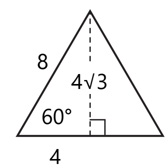


leg of right triangle = side length = 36/4 = 9

hypotenuse of right triangle = $\sqrt{2}$ (leg) = $9\sqrt{2}$

diagonal = hypotenuse of right triangle = $9\sqrt{2}$ in

4. An altitude divides an equilateral triangle into two congruent 30-60-90 triangles. In each 30-60-90 triangle, the longer leg is the altitude of the equilateral triangle, and the shorter leg is half the side length of the equilateral triangle.

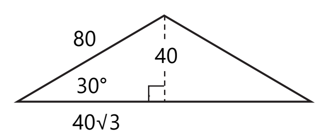


shorter leg of right triangle = half the side length = 4

longer leg of right triangle = $\sqrt{3}$ (shorter leg) = $4\sqrt{3}$

altitude = longer leg of right triangle = $4\sqrt{3}$ cm

5. In an isosceles triangle, the median to the base is also an altitude and an angle bisector. In an isosceles triangle with base angle 30°, the median to the base divides the triangle into two congruent 30-60-90 triangles. In each 30-60-90 triangle, the shorter leg is the median of the isosceles triangle, and the longer leg is half the base of the isosceles triangle.



shorter leg of right triangle = median = 40

longer leg of right triangle = $\sqrt{3}$ (shorter leg) = $40\sqrt{3}$

hypotenuse of right triangle = 2 (shorter leg) = 80

perimeter = 2 (hypotenuse) + 2 (longer leg)

= 160 + $80\sqrt{3}$ mm

LESSON 78

1. $\sin 50° = \dfrac{a}{8}$ and $\sin 40° = \dfrac{a}{b} = \dfrac{6.1}{b}$

$a = 8 \sin 50°$

$a \approx 6.1$

$b \sin 40° = 6.1$

$b = \dfrac{6.1}{\sin 40°}$

$b \approx 9.5$

2. $\tan 68° = \frac{a}{5}$ and $\tan 74° = \frac{a}{b} = \frac{12.4}{b}$

a = 5 tan 68°

a ≈ 12.4

b tan 74° = 12.4

$b = \frac{12.4}{\tan 74°}$

b ≈ 3.6

3. $\sin 26° = \frac{3}{a}$ and $\sin 39° = \frac{a}{b} = \frac{6.8}{b}$

a sin 26° = 3

$a = \frac{3}{\sin 26°}$

a ≈ 6.8

b sin 39° = 6.8

$b = \frac{6.8}{\sin 39°}$

b ≈ 10.8

LESSON 79

1. $\sin 50° = \frac{x}{6}$ and $\sin\theta = \frac{x}{7.3} = \frac{4.6}{7.3}$

x = 6 sin 50°

x ≈ 4.6

$\theta = \sin^{-1}$ (4.6/7.3)

θ ≈ 39.1°

2. $\tan 67° = \frac{x}{4}$ and $\tan\theta = \frac{x}{3} = \frac{9.4}{3}$

x = 4 tan 67°

x ≈ 9.4

$\theta = \tan^{-1}$ (9.4/3)

θ ≈ 72.3°

3. $\tan 25° = \frac{x}{8}$ and $\tan\theta = \frac{8}{11+x} = \frac{8}{14.7}$

x = 8 tan 25°

x ≈ 3.7

$\theta = \tan^{-1}$ (8/14.7)

θ ≈ 28.6°

LESSON 80

1 ~ 2. *Answers may vary slightly due to rounding.*

1. a. BD = 10 sin 54° ≈ 8.1

AD = 10 cos 54° ≈ 5.9

$m\angle ABD$ = 90 – 54 = 36°

b. BD ≈ 8.1 (Found in Problem 1a.)

$m\angle C$ ≈ $\tan^{-1}$ (8.1/10) ≈ 39°

$m\angle CBD$ ≈ 90 – 39 = 51°

BC = 10 / sin 51° ≈ 12.9

c. BC ≈ 12.9 (Found in Problem 1b.)

$AC = AD + DC$ ≈ 5.9 + 10 = 15.9

$m\angle C$ ≈ 39° (Found in Problem 1b.)

$m\angle ABC = m\angle ABD + m\angle CBD$ ≈ 36 + 51 = 87°

2. a. $AC = \sqrt{14^2 + 4^2} = \sqrt{212}$ ≈ 14.6

$m\angle CAD$ = $\tan^{-1}$ (4/14) ≈ 15.9°

$m\angle ACD$ ≈ 90 – 15.9 = 74.1°

b. $BD = \sqrt{16^2 - 14^2} = \sqrt{60}$ ≈ 7.7

$m\angle BAD$ = $\cos^{-1}$ (14/16) ≈ 29°

$m\angle B$ ≈ 90 – 29 = 61°

c. $BC = BD - CD$ ≈ 7.7 – 4 = 3.7

AC ≈ 14.6 (Found in Problem 2a.)

$m\angle B$ ≈ 61° (Found in Problem 2b.)

$m\angle BCA$ ≈ 180 – $m\angle ACD$ = 180 – 74.1 = 105.9°

$m\angle BAC = m\angle BAD - m\angle CAD$ ≈ 29 – 15.9 = 13.1°

LESSON 81

1. height = 6 sin 60° ≈ 5.2

area = $\frac{1}{2}$(6)(6) sin 60° ≈ 15.6

2. side length = 16/4 = 4

area = (4)(4) sin 40° ≈ 10.3

3. Opposite sides of a parallelogram are congruent, so the parallelogram has side lengths of 10 and 5.

area = (10)(5) sin 110° ≈ 47

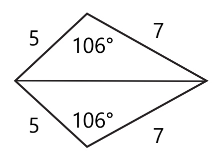
4. See the kite on the right.

perimeter = 2(5) + 2(7) = 24

area = 2 congruent triangles

= 2(1/2)(5)(7) sin 106°

≈ 33.6



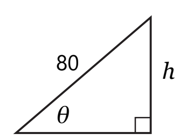
LESSON 82

1 ~ 2. *Diagrams are not drawn to scale.*

1. max h = 80 sin 65° ≈ 72.5

min h = 80 sin 40° ≈ 51.4

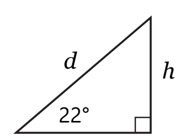
The maximum height is about 72.5 ft and the minimum height is about 51.4 ft.



2. d = (100 ft/s)(5 sec) = 500 ft

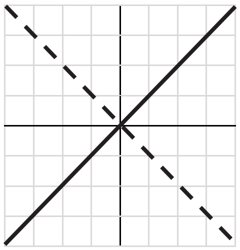
$h = d$ sin 22° ≈ 187.3

The plane is about 187.3 ft above the ground.

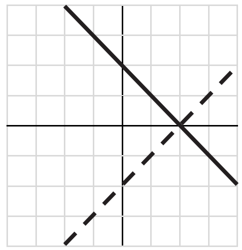


LESSON 84

1. $y = -x$

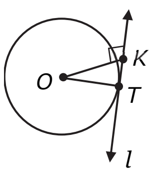


2. (2, 0)



LESSON 91

1.



Assume that l is not perpendicular to $\overline{OT}$. Then there must be a point K on l such that $l \perp \overline{OK}$.

By the definition of tangent, tangent line l intersects circle O at only T. This means that K is outside of circle O and thus $\overline{OK}$ is longer than radius $\overline{OT}$ ($OT < OK$).

But look at $\triangle OTK$. Because $l \perp \overline{OK}$, it is a right triangle with hypotenuse $\overline{OT}$. The hypotenuse is the longest side of a right triangle, meaning that $OT > OK$.

$OT < OK$ and $OT > OK$ cannot be both true. Our assumption leads to a contradiction and must be false. Therefore, l must be perpendicular to $\overline{OT}$.

2. Assume that l is not tangent to circle O. Then l must intersect circle O at another point K besides T.

Both T and K are on circle O, so $OT = OK$ as radii.

But look at $\triangle OTK$. Because $l \perp \overline{OT}$, it is a right triangle with hypotenuse $\overline{OK}$. The hypotenuse is the longest side of a right triangle, meaning that $OT < OK$.

$OT = OK$ and $OT < OK$ cannot be both true. Our assumption leads to a contradiction and must be false. Therefore, l must be tangent to circle O.

3. Statements (Reasons)

1. $\overline{CA}$ and $\overline{CB}$ are tangents to circle P. (Given)
2. Draw $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. (Construction)
3. $\overline{PA} \perp \overline{AC}$, $\overline{PB} \perp \overline{BC}$ (Tangent and radius are perpendicular. See Theorem 91.1.)
4. $\angle CAP$ and $\angle CBP$ are right angles. (Def. of perpendicular)
5. $\triangle CAP$ and $\triangle CBP$ are right triangles. (Def. of right triangle)
6. $\overline{AP} \cong \overline{BP}$ (All radii of a circle are congruent.)
7. $\overline{CP} \cong \overline{CP}$ (Reflexive Property)
8. $\triangle CAP \sim \triangle CBP$ (HL)
9. $\overline{CA} \cong \overline{CB}$ (CPCTC)

LESSON 92

1. Statements (Reasons)

1. $\widehat{DE} \cong \widehat{FG}$ (Given)
2. $m\widehat{DE} = m\widehat{FG}$ (Def. of congruent arcs)
3. $m\widehat{DE} = m\angle DPE$, $m\widehat{FG} = m\angle FPG$ (Def. of arc measure)
4. $m\angle DPE = m\angle FPG$ (Substitution Property)
5. $\angle DPE \cong \angle FPG$ (Def. of congruent angles)

2. Statements (Reasons)

1. $\angle DPE \cong \angle FPG$ (Given)
2. $m\angle DPE = m\angle FPG$ (Def. of congruent angles)
3. $m\angle DPE = m\widehat{DE}$, $m\angle FPG = m\widehat{FG}$ (Def. of arc measure)
4. $m\widehat{DE} = m\widehat{FG}$ (Substitution Property)
5. $\widehat{DE} \cong \widehat{FG}$ (Def. of congruent arcs)

LESSON 93

1. Statements (Reasons)

1. $\widehat{AB} \cong \widehat{CD}$ (Given)
2. $\angle AOB \cong \angle COD$ (Congruent arcs have congruent central angles. See Theorem 92.1)
3. $\overline{OA} \cong \overline{OB} \cong \overline{OC} \cong \overline{OD}$ (All radii of a circle are ≅.)
4. $\triangle AOB \cong \triangle COD$ (SAS)
5. $\overline{AB} \cong \overline{CD}$ (CPCTC)

2. Statements (Reasons)

1. $\overline{AB} \cong \overline{CD}$ (Given)
2. $\overline{OA} \cong \overline{OB} \cong \overline{OC} \cong \overline{OD}$ (All radii of a circle are ≅.)
3. $\triangle AOB \cong \triangle COD$ (SSS)
4. $\angle AOB \cong \angle COD$ (CPCTC)
5. $\widehat{AB} \cong \widehat{CD}$ (Congruent central angles have congruent arcs. See Theorem 92.1)

3. Statements (Reasons)

1. $\overline{PG} \perp \overline{EF}$ (Given)
2. $\angle PHE$ are $\angle PHF$ right ∠s. (Def. of perpendicular)
3. $\triangle PHE$ and $\triangle PHF$ are right △s. (Def. of right △)
4. $\overline{PE} \cong \overline{PF}$ (All radii of a circle are ≅.)
5. $\overline{PH} \cong \overline{PH}$ (Reflexive Property)
6. $\triangle PHE \cong \triangle PHF$ (HL)
7. $\overline{HE} \cong \overline{HF}$ (CPCTC)
8. $\angle GPE \cong \angle GPF$ (CPCTC)
9. $\widehat{GE} \cong \widehat{GF}$ (Congruent central angles have congruent arcs. See Theorem 92.1)

4. Statements (Reasons)
   1. $\overline{HE} \cong \overline{HF}$ (Given)
   2. $\overline{PE} \cong \overline{PF}$ (All radii of a circle are ≅.)
   3. $\overline{PH} \cong \overline{PH}$ (Reflexive Property)
   4. $\triangle PHE \cong \triangle PHF$ (SSS)
   5. $\angle PHE \cong \angle PHF$ (CPCTC)
   6. $\angle PHE$ & $\angle PHF$ are a linear pair. (Def. of linear pair)
   7. $\overline{PG} \perp \overline{EF}$ (Two lines intersecting to form a linear pair of congruent angles are perpendicular. See Theorem 31.3.)

LESSON 94

1. Statements (Reasons)
   1. $\overline{OK} \perp \overline{ST}$, $\overline{OL} \perp \overline{UV}$, $\overline{ST} \cong \overline{UV}$ (Given)
   2. $\overline{KS} \cong \overline{KT}$, $\overline{LU} \cong \overline{LV}$ (A radius perpendicular to a chord bisects the chord. See Theorem 93.2.)
   3. $\overline{KT} \cong \overline{LV}$ (Halves of congruent segments are congruent. You could prove this statement in more detailed steps as shown below.)
      a. $ST = UV$ (Def. of congruent segments, Step 1)
      b. $ST = 2KT$, $UV = 2LV$ (Def. of bisect)
      c. $2KT = 2LV$ (Substitution Property)
      d. $KT = LV$ (Division Property)
      e. $\overline{KT} \cong \overline{LV}$ (Def. of congruent segments)
   4. Draw $\overline{OT}$ and $\overline{OV}$. (Construction)
   5. $\overline{OT} \cong \overline{OV}$ (All radii of a circle are congruent.)
   6. $\triangle OKT \cong \triangle OLV$ (HL, Steps 3 and 5)
   7. $\overline{OK} \cong \overline{OL}$ (CPCTC)

2. Statements (Reasons)
   1. $\overline{OK} \perp \overline{ST}$, $\overline{OL} \perp \overline{UV}$, $\overline{OK} \cong \overline{OL}$ (Given)
   2. Draw $\overline{OT}$ and $\overline{OV}$. (Construction)
   3. $\overline{OT} \cong \overline{OV}$ (All radii of a circle are congruent.)
   4. $\triangle OKT \cong \triangle OLV$ (HL, Steps 1 and 3)
   5. $\overline{KT} \cong \overline{LV}$ (CPCTC)
   6. $\overline{KS} \cong \overline{KT}$, $\overline{LU} \cong \overline{LV}$ (A radius perpendicular to a chord bisects the chord. See Theorem 93.2.)
   8. $\overline{ST} \cong \overline{UV}$ (Multiples of congruent segments are congruent. You could prove this statement in more detailed steps as shown below.)
      a. $KT = LV$ (Def. of congruent segments, Step 5)
      b. $2KT = 2LV$ (Multiplication Property)
      b. $ST = 2KT$, $UV = 2LV$ (Def. of bisect)
      c. $ST = UV$ (Substitution Property)
      d. $\overline{ST} \cong \overline{UV}$ (Def. of congruent segments)

3. Statements (Reasons)
   1. $\overline{AD} \cong \overline{DC}$ (Given)
   2. $\overline{OB}$ bisects $\overline{AC}$. (Def. of bisect)
   3. $\widehat{AB} \cong \widehat{BC}$ (A radius perpendicular to a chord bisects the arc of that chord. See Theorem 93.2.)
   4. $\overline{AB} \cong \overline{BC}$ (Congruent arcs have congruent chords. See Theorem 93.1.)

LESSON 95

1. Statements (Reasons)
   1. inscribed $\angle ABC$, diameter $\overline{BC}$ (Given)
   2. $\overline{OA} \cong \overline{OB}$ (All radii of a circle are congruent.)
   3. $\angle A \cong \angle B$ (Base Angles Theorem [40.1])
   4. $m\angle A = m\angle B$ (Def. of congruent angles)
   5. $m\angle AOC = m\angle A + m\angle B$ (Triangle Exterior Angle Theorem [32.2])
   6. $m\angle AOC = m\angle B + m\angle B$ (Substitution)
   7. $m\angle AOC = 2m\angle B$ (Simplify.)
   8. $m\widehat{AC} = m\angle AOC$ (Def. of arc measure)
   9. $m\widehat{AC} = 2m\angle B$ (Transitive Property)

2. Statements (Reasons)
   1. inscribed $\angle ABC$, diameter $\overline{DB}$ (Given)
   2. $m\widehat{AC} = m\widehat{AD} + m\widehat{DC}$ (Arc Addition Postulate)
   3. $m\widehat{AD} = 2m\angle ABD$, $m\widehat{DC} = 2m\angle DBC$ (Proved in Problem 1.)
   4. $m\widehat{AC} = 2m\angle ABD + 2m\angle DBC$ (Substitution)
   5. $m\widehat{AC} = 2(m\angle ABD + m\angle DBC)$ (Distributive Prop.)
   5. $m\angle ABD + m\angle DBC = m\angle B$ (Angle Addition Post.)
   6. $m\widehat{AC} = 2m\angle B$ (Substitution)

3. Statements (Reasons)
   1. inscribed $\angle ABC$, diameter $\overline{DB}$ (Given)
   2. $m\widehat{AC} = m\widehat{AD} - m\widehat{CD}$ (Arc Addition Postulate)
   3. $m\widehat{AD} = 2m\angle ABD$, $m\widehat{CD} = 2m\angle CBD$ (Proved in Problem 1.)
   4. $m\widehat{AC} = 2m\angle ABD - 2m\angle CBD$ (Substitution)
   5. $m\widehat{AC} = 2(m\angle ABD - m\angle CBD)$ (Distributive Prop.)
   5. $m\angle ABD - m\angle CBD = m\angle B$ (Angle Addition Post.)
   6. $m\widehat{AC} = 2m\angle B$ (Substitution)

4. Statements (Reasons)
   1. inscribed $\angle DGE$ and $\angle DFE$ (Given)
   2. $m\angle G = m\widehat{DE}/2$, $m\angle F = m\widehat{DE}/2$ (Theorem 95.1)
   3. $m\angle G = m\angle F$ (Transitive Property)
   4. $\angle G \cong \angle F$ (Def. of congruent angles)

5. Statements (Reasons)
    1. inscribed $\angle DPE$ and $\angle FQG$, $\widehat{DE} \cong \widehat{FG}$ (Given)
    2. $m\widehat{DE} = m\widehat{FG}$ (Def. of congruent arcs)
    3. $m\widehat{DE} = 2m\angle P$, $m\widehat{FG} = 2m\angle Q$ (Theorem 95.1)
    4. $2m\angle P = 2m\angle Q$ (Transitive Property)
    5. $m\angle P = m\angle Q$ (Division Property)
    6. $\angle P \cong \angle Q$ (Def. of congruent angles)

LESSON 96

1. $\widehat{ADC}$ is the intercepted arc of $\angle B$. An intercepted arc measures twice its inscribed angle, so $m\widehat{ADC} = 2m\angle B = 2(90) = 180°$.

   By definition, a semicircle is an arc whose measure is 180° and whose endpoints are the endpoints of a diameter. So, $\widehat{ADC}$ is a semicircle and $\overline{AC}$ is a diameter.
2. A diameter divides a circle into two semicircles, so $\widehat{ADC}$ is a semicircle and $m\widehat{ADC} = 180°$. $\angle B$ is an inscribed angle of $\widehat{ADC}$. An inscribed angle measures half its intercepted arc, so $m\angle B = m\widehat{ADC}/2 = 180/2 = 90°$.
3. 2. An intercepted arc measures twice its inscribed angle. See Theorem 95.1.
    4. Substitution Property
    5. Division Property

LESSON 97

1. Statements (Reasons)
    1. tangent $\overleftrightarrow{BP}$, diameter $\overline{AB}$ (Given)
    2. $m\angle ABP = 90°$ (Tangent and radius are perpendicular. See Theorem 91.1)
    3. $\widehat{ACB}$ is a semicircle and $m\widehat{ACB} = 180°$. (Def. of semicircle)
    4. $m\widehat{ACB}/2 = 90°$ (Division Property)
    5. $m\angle ABP = m\widehat{ACB}/2$ (Transitive Property)
2. Statements (Reasons)
    1. tangent $\overleftrightarrow{BP}$, acute $\angle ABP$ (Given)
    2. Draw diameter $\overline{BC}$. (Construction)
    3. $m\angle CBP = m\widehat{CAB}/2$ (Proved in Problem 1.)
    4. $m\angle CBA = m\widehat{CA}/2$ (Inscribed Angle Theorem [95.1])
    5. $m\angle ABP = m\angle CBP - m\angle CBA$ (Angle Addition Post.)
    6. $m\angle ABP = m\widehat{CAB}/2 - m\widehat{CA}/2$ (Substitution Prop.)
    7. $m\angle ABP = (m\widehat{CAB} - m\widehat{CA})/2$ (Distributive Prop.)
    8. $m\widehat{CAB} - m\widehat{CA} = m\widehat{AB}$ (Arc Addition Post.)
    9. $m\angle ABP = m\widehat{AB}/2$ (Substitution Prop.)
3. Statements (Reasons)
    1. tangent $\overleftrightarrow{BP}$, acute $\angle ABP$ (Given)
    2. Draw diameter $\overline{BC}$. (Construction)
    3. $m\angle CBP = m\widehat{CDB}/2$ (Proved in Problem 1.)
    4. $m\angle ABC = m\widehat{AC}/2$ (Inscribed Angle Theorem [95.1])
    5. $m\angle ABP = m\angle ABC + m\angle CBP$ (Angle Addition Post.)
    6. $m\angle ABP = m\widehat{AC}/2 + m\widehat{CDB}/2$ (Substitution Prop.)
    7. $m\angle ABP = (m\widehat{AC} + m\widehat{CDB})/2$ (Distributive Prop.)
    8. $m\widehat{AC} + m\widehat{CDB} = m\widehat{ACB}$ (Arc Addition Post.)
    9. $m\angle ABP = m\widehat{ACB}/2$ (Substitution Prop.)
4. Statements (Reasons)
    1. chords $\overline{DF}$ and $\overline{EG}$ (Given)
    2. Draw $\overline{DG}$. (Construction)
    3. $m\angle G = m\widehat{DE}/2$, $m\angle D = m\widehat{FG}/2$ (Inscribed Angle Theorem [95.1])
    4. $m\angle 1 = m\angle G + m\angle D$ (Triangle Exterior Angle Theorem [32.2])
    5. $m\angle 1 = m\widehat{DE}/2 + m\widehat{FG}/2$ (Substitution Property)
    6. $m\angle 1 = (m\widehat{DE} + m\widehat{FG})/2$ (Distributive Property)

LESSON 98

1. Statements (Reasons)
    1. two secants (Given)
    2. $m\angle 3 = m\angle 1 + m\angle 2$ (Triangle Exterior Angle Theorem [32.2])
    3. $m\angle 1 = m\angle 3 - m\angle 2$ (Subtraction Property)
    4. $m\angle 3 = m\widehat{CD}/2$, $m\angle 2 = m\widehat{AB}/2$ (Inscribed Angle Theorem [95.1])
    5. $m\angle 1 = m\widehat{CD}/2 - m\widehat{AB}/2$ (Substitution Property)
    6. $m\angle 1 = (m\widehat{CD} - m\widehat{AB})/2$ (Distributive Property)
2. Statements (Reasons)
    1. a secant, a tangent (Given)
    2. $m\angle 3 = m\angle 1 + m\angle 2$ (Triangle Exterior Angle Theorem [32.2])
    3. $m\angle 1 = m\angle 3 - m\angle 2$ (Subtraction Property)
    4. $m\angle 3 = m\widehat{CB}/2$ (Chord-Tangent Angle Theorem [97.1])
    5. $m\angle 2 = m\widehat{AB}/2$ (Inscribed Angle Theorem [95.1])
    6. $m\angle 1 = m\widehat{CB}/2 - m\widehat{AB}/2$ (Substitution Property)
    7. $m\angle 1 = (m\widehat{CB} - m\widehat{AB})/2$ (Distributive Property)

3. Statements (Reasons)
   1. two tangents (Given)
   2. $m\angle 3 = m\angle 1 + m\angle 2$ (Triangle Exterior Angle Theorem [32.2])
   3. $m\angle 1 = m\angle 3 - m\angle 2$ (Subtraction Property)
   4. $m\angle 3 = m\widehat{ACB}/2$, $m\angle 2 = m\widehat{AB}/2$ (Chord-Tangent Angle Theorem [97.1])
   5. $m\angle 1 = m\widehat{ACB}/2 - m\widehat{AB}/2$ (Substitution Property)
   6. $m\angle 1 = (m\widehat{ACB} - m\widehat{AB})/2$ (Distributive Property)

LESSON 99

1. Statements (Reasons)
   1. two chords (Given)
   2. $\angle A \cong \angle D$, $\angle C \cong \angle B$ (Inscribed angles of the same arc are congruent. See Theorem 95.2.)
   3. $\triangle PAC \sim \triangle PDB$ (AA)
   4. $\frac{PA}{PD} = \frac{PC}{PB}$ (Corresponding sides of similar triangles are proportional.)
   5. $PA \cdot PB = PC \cdot PD$ (Cross multiply.)
2. Statements (Reasons)
   1. two secants (Given)
   2. $\angle P \cong \angle P$ (Reflexive Property)
   3. $\angle B \cong \angle D$ (Inscribed angles of the same arc are congruent. See Theorem 95.2.)
   4. $\triangle PBC \sim \triangle PDA$ (AA)
   5. $\frac{PB}{PD} = \frac{PC}{PA}$ (Corresponding sides of similar triangles are proportional.)
   6. $PA \cdot PB = PC \cdot PD$ (Cross multiply.)
3. Statements (Reasons)
   1. a tangent, a secant (Given)
   2. $\angle P \cong \angle P$ (Reflexive Property)
   3. $m\angle ACP = m\widehat{AC}/2$ (A chord-tangent angle measures half its intercepted arc. See Theorem 97.1.)
   3. $m\angle ABC = m\widehat{AC}/2$ (Inscribed Angle Theorem [95.1])
   5. $m\angle ACP = m\angle ABC$ (Transitive Property)
   6. $\angle ACP \cong \angle ABC$ (Def. of congruent angles)
   4. $\triangle PAC \sim \triangle PCB$ (AA)
   5. $\frac{PA}{PC} = \frac{PC}{PB}$ (Corresponding sides of similar triangles are proportional.)
   6. $PA \cdot PB = PC^2$ (Cross multiply.)

LESSON 100

1. Congruent chords have congruent arcs. Let $m\widehat{AD} = m\widehat{BC} = x$. A circle has 360°, so $112 + 56 + 2x = 360°$. Solve the equation to get $x = m\widehat{AD} = m\widehat{BC} = 96°$.

   $m\angle 1 = m\widehat{AB}/2 = 56°$ (inscribed angle)

   $m\angle 2 = m\widehat{CD}/2 = 28°$ (inscribed angle)

   $m\angle 3 = m\widehat{CD}/2 = 28°$ (inscribed angle)

   $m\angle 4 = m\widehat{AD}/2 = 48°$ (inscribed angle)

   $m\angle 5 = m\widehat{BC}/2 = 48°$ (inscribed angle)

   $m\angle 6 = (m\widehat{AB} + m\widehat{CD})/2 = 84°$ (chord-chord angle)

   $m\angle 7 = 180 - m\angle 6 = 96°$ (supplementary angles)

   $m\angle 8 = (m\widehat{AB} - m\widehat{CD})/2 = 28°$ (secant-secant angle)

2. $m\widehat{BC} = 180 - m\widehat{AB} = 180 - 70 = 110°$

   $m\widehat{AE} = 180 - m\widehat{CD} - m\widehat{DE} = 180 - 70 - 38 = 72°$

   $m\angle 1 = m\widehat{AB}/2 = 35°$ (inscribed angle)

   $m\angle 2 = m\widehat{BC}/2 = 55°$ (inscribed angle)

   $m\angle 3 = (m\widehat{CD} + m\widehat{DE})/2 = 54°$ (inscribed angle)

   $m\angle 4 = m\widehat{AE} + m\widehat{DE} = 110°$ (central angle)

   $m\angle 5 = m\widehat{BC}/2 = 55°$ (chord-tangent angle)

   $m\angle 6 = (m\widehat{BEC} - m\widehat{BC})/2 = 70°$ (tangent-tangent angle)

   $m\angle 7 = 90°$ (tangent-radius angle)

   $m\angle 8 = (m\widehat{AB} - m\widehat{DE})/2 = 16°$ (secant-secant angle)

LESSON 101

1. Statements (Reasons)
   1. tangent $\overline{AB}$, tangent $\overline{AC}$ (Given)
   2. $\overline{AB} \cong \overline{AC}$ (Tangent segments to a circle from a point are congruent. See Theorem 91.2.)
   3. $\overline{OB} \cong \overline{OC}$ (All radii of a circle are congruent.)
   4. $\overline{OA} \cong \overline{OA}$ (Reflexive Property)
   5. $\triangle OAB \sim \triangle OAC$ (SSS)
   6. $\angle OAB \cong \angle OAC$ (CPCTC)
2. Statements (Reasons)
   1. $\widehat{PQ} \cong \widehat{SR}$ (Given)
   2. $\overline{PQ} \cong \overline{SR}$ (Congruent arcs have congruent chords. See Theorem 93.1.)
   3. $\angle P \cong \angle S$, $\angle Q \cong \angle R$ (Inscribed angles of the same arc are congruent. See Theorem 95.2.)
   4. $\triangle PQT \cong \triangle SRT$ (ASA)

3. Statements (Reasons)

1. $\overline{AB} \parallel \overline{CD}$ (Given)
2. Draw $\overline{BC}$. (Construction)
3. $\angle ABC \cong \angle BCD$ (Alternate interior ∠s are ≅.)
4. $m\angle ABC = m\angle BCD$ (Def. of congruent angles)
5. $m\angle ABC = m\widehat{AC}/2$, $m\angle BCD = m\widehat{BD}/2$ (An inscribed angle measures half its intercepted arc. See Theorem 95.1.)
6. $m\widehat{AC}/2 = m\widehat{BD}/2$ (Substitution Property)
7. $m\widehat{AC} = m\widehat{BD}$ (Multiplication Property)
8. $\widehat{AC} \cong \widehat{BD}$ (Def. of congruent arcs)

LESSON 106

1 ~ 4. *Answers may vary slightly due to rounding.*

1. circumference of wheel = $\pi d \approx (22/7)(80) \approx 251$ cm

distance = 5,000 × circumference of wheel

≈ 5,000 × 251 = 1,255,000 cm = 12.55 km

The car will travel about 13 km.

2. circumference of wheel = $\pi d \approx (22/7)(75) \approx 236$ cm

revolutions = distance / circumference

≈ 200,000 cm / 236 cm ≈ 847

The wheel will make about 847 revolutions.

3. length = circumference of a circle with diameter 100 + two sides of a square with side length 100

$= 100\pi + 2(100) \approx 100(22/7) + 2(100) \approx 514$

The length of the track is about 514 meters.

4. time = distance / speed

= (514)(4)/2 = 1028 seconds ≈ 17 minutes

It will take about 17 minutes.

LESSON 113

1. $\dfrac{\text{sector area}}{\text{circle area}} = \dfrac{\text{arc length}}{\text{circumference}}$

$\dfrac{x}{81\pi} = \dfrac{6\pi}{18\pi} = \dfrac{1}{3} \rightarrow 3x = 81\pi \rightarrow x = 27\pi$

2. LA = n triangles with base s and height l

$= n \cdot \frac{1}{2}sl = \frac{1}{2}nsl = \frac{1}{2}Pl$

3. s = circumference of base = $2\pi r$

LA = area of a sector with radius l and arc length s

$\dfrac{\text{sector area } LA}{\text{circle area}} = \dfrac{\text{arc length } s}{\text{circumference}}$

$\dfrac{LA}{\pi l^2} = \dfrac{2\pi r}{2\pi l} \rightarrow LA = \dfrac{2\pi r}{2\pi l} \cdot \pi l^2 = \pi r l$

LESSON 115

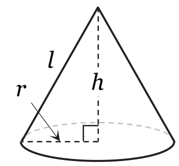
1. Use the surface area to find the slant height.

$SA = \pi r^2 + \pi r l = \pi(7)^2 + \pi(7)l = 224\pi$

$49 + 7l = 224$

$l = 25$

Use the Pythagorean Theorem or the 7-24-25 Pythagorean triple to find $h = 24$.



$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(7)^2(24) = 392\pi$

So, the volume is 392π cm^3.

LESSON 116

1. The cylinder has radius r and height $2r$.

SA = lateral area of the cylinder

= (base circumference)(height)

$= (2\pi r)(2r) = 4\pi r^2$

2. sphere V = cylinder V – cone V

$V = \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \pi r^2(2r) - \frac{1}{3}\pi r^2(2r) = 2\pi r^3 - \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$

LESSON 122

1. distance from Morgan's house to the library

$= \sqrt{(4-0)^2 + (7-4)^2} = \sqrt{25} = 5$ km

time = distance/speed = 5/10 = 0.5 hour

So, it took half an hour (or 30 minutes).

LESSON 125

1. a. (0, 3)

b. original slope = –2

perpendicular slope = 1/2

point-slope form: $y - 3 = (1/2)(x - 0)$

slope-intercept form: $y = (1/2)x + 3$

c. Solve the system of $2x + y = 8$ and $y = (1/2)x + 3$.

$2x + (1/2)x + 3 = 8$

$4x + x + 6 = 16$ (Multiply both sides by 2.)

$x = 2$

$y = (1/2)(2) + 3 = 4$

The lines intersect at (2, 4).

d. distance between (0, 3) and (2, 4)

$= \sqrt{(2-0)^2 + (4-3)^2} = \sqrt{5}$

2. a. The y-intercept of $y = -x + 2$ is (0, 2).

b. Find the line perpendicular to $x + y = 6$ passing through (0, 2).

original slope = −1

perpendicular slope = 1

point-slope form: $y - 2 = x - 0$

slope-intercept form: $y = x + 2$

c. Find the intersection between $x + y = 6$ and $y = x + 2$.

$x + (x + 2) = 6$

$x = 2$

$y = 2 + 2 = 4$

The lines intersect at (2, 4).

d. Find the distance between (0, 2) and (2, 4).

$d = \sqrt{(2-0)^2 + (4-2)^2} = 2\sqrt{2}$

3. a. The y-intercept of $y = 2x - 1$ is (0, −1).

b. Find the line perpendicular to $2x - y = 6$ passing through (0, −1).

original slope = 2

perpendicular slope = −1/2

point-slope form: $y - (-1) = (-1/2)(x - 0)$

slope-intercept form: $y = (-1/2)x - 1$

c. Find the intersection between $2x - y = 6$ and $y = (-1/2)x - 1$.

$2x - ((-1/2)x - 1) = 6$

$2x + (1/2)x + 1 = 6$

$4x + x + 2 = 12$ (Multiply both sides by 2.)

$x = 2$

$y = (-1/2)(2) - 1 = -2$

The lines intersect at (2, −2).

d. Find the distance between (0, −1) and (2, −2).

$d = \sqrt{(2-0)^2 + (-2-(-1))^2} = \sqrt{5}$

LESSON 128

1. a. The slope of $\overline{CP}$ is 1/2.

b. The tangent line is perpendicular to $\overline{CP}$, so the slope of the tangent line is −2.

c. The tangent line has slope −2 and passes through $P(4, 2)$, so the point-slope form is $y - 2 = -2(x - 4)$.

2. a. $y = x + 2$

b. $(x + 2)^2 + (x + 2)^2 = 8$

$2(x + 2)^2 = 8$

$(x + 2)^2 = 4$

$x + 2 = 2$, $x + 2 = -2$

$x = 0$, $x = -4$

2. c. If $x = 0$, then $y = 0 + 2 = 2$.

If $x = -4$, then $y = -4 + 2 = -2$.

So, they intersect at (0, 2) and (−4, −2).

3. The slope of $\overline{CP}$ is −1/3, meaning that the tangent line has slope 3 and passes through (3, 0). So, the point-slope form of the tangent line is $y = 3(x - 3)$.

4. $(x - 1)^2 + (-x + 4 - 2)^2 = 13$

$2x^2 - 6x - 8 = 0$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x = 4$, $x = -1$

If $x = 4$, then $y = -4 + 4 = 0$.

If $x = -1$, then $y = -(-1) + 4 = 5$.

So, they intersect at (4, 0) and (−1, 5).

LESSON 130

1. $E(2b, 2h)$

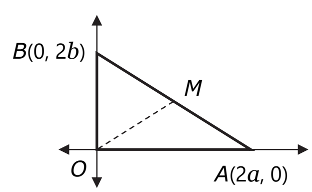
2. $P(b, h)$, $Q(3b, h)$, $R(2b, 0)$

3. $\overline{OD}$ is horizontal because it is on the x-axis. $\overline{ER}$ is vertical because E and R have the same x-coordinate. So, $\overline{OD}$ and $\overline{ER}$ are perpendicular to each other.

4 $OQ = \sqrt{(3b-0)^2 + (h-0)^2} = \sqrt{9b^2 + h^2}$

$DP = \sqrt{(b-4b)^2 + (h-0)^2} = \sqrt{9b^2 + h^2}$

The two segments have the same length.

5. 

Place a right triangle with legs $2a$ and $2b$ on a coordinate plane.

Let M be the midpoint of $\overline{AB}$. By the midpoint formula,

$M = \left(\frac{2a+0}{2}, \frac{0+2b}{2}\right) = (a\ , b)$

By the distance formula,

$MO = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

$MA = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

$MB = \sqrt{(0-a)^2 + (2b-b)^2} = \sqrt{a^2 + b^2}$

$MO = MA = MB$, so M is equidistant from each vertex.

LESSON 136

3. ASA

LESSON 137

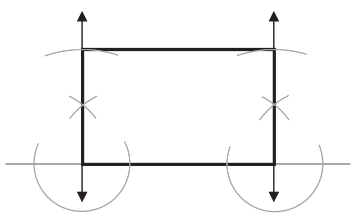
1. Construct the perpendicular bisector of each side of the triangle. If your construction is correct, the perpendicular bisectors should meet in one point. That point is the circumcenter of your triangle.

2. If your construction is correct, the circumcenter should be equidistant from all three vertices of your triangle.

3. If your construction is correct, the circumcenter should be the midpoint of the hypotenuse of your triangle.

LESSON 138

1.



LESSON 142

1. $\overline{QJ} \cong \overline{RK}$ as radii of congruent circles. $\overline{QJ} \perp \overline{QR}$ and $\overline{RK} \perp \overline{QR}$ by construction. $\overline{QJ} \parallel \overline{RK}$ because they are perpendicular to the same line. One pair of sides is congruent and parallel, so $QRKJ$ is a parallelogram. Opposite angles of a parallelogram are congruent, so $\angle QJK$ and $\angle RKJ$ are also right angles.

LESSON 146

1. $P(2) \times P(3)$

$= \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$

2. $P(8) \times P(8)$

$= \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$

3. P(1st digit < 4) ×
P(2nd digit any)

$= \frac{3}{9} \times \frac{9}{9} = \frac{1}{3}$

4. P(1st digit any) ×
P(2nd digit = 5)

$= \frac{9}{9} \times \frac{1}{9} = \frac{1}{9}$

LESSON 147

1. $P(2) \times P(3|2)$

$= \frac{1}{9} \times \frac{1}{8} = \frac{1}{72}$

2. $P(8) \times P(8|8)$

$= \frac{1}{9} \times \frac{0}{8} = 0$

3. P(1st digit < 4) ×
P(2nd digit any|1st < 4)

$= \frac{3}{9} \times \frac{8}{8} = \frac{1}{3}$

4. P(2nd digit = 5) ×
P(1st digit any|2nd = 5)

$= \frac{1}{9} \times \frac{8}{8} = \frac{1}{9}$

LESSON 148

1. $P = 36/81 = 4/9$
9 × 9 = 81 possible
4 × 9 = 36 favorable:
10s place: 1, 2, 3, 4
1s place: 1 to 9

2. $P = 45/81 = 5/9$
9 × 9 = 81 possible
9 × 5 = 45 favorable:
10s place: 1 to 9
1s place: 1, 3, 5, 7, 9

3. P(1st digit < 5) × P(2nd digit odd)

$= \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$

4. Use the probabilities from Problems 1 through 3.
P(less than 50) + P(odd) − P(less than 50 and odd)

$= \frac{4}{9} + \frac{5}{9} - \frac{20}{81} = \frac{61}{81}$

LESSON 149

1. P(odd) × P(odd) × P(odd) × P(odd) × P(odd)

$= \frac{3}{6} \times \frac{3}{6} \times \frac{3}{6} \times \frac{3}{6} \times \frac{3}{6} = \frac{1}{32}$

2. $P(x$ is odd and y is odd$) = P(x$ is odd$) \times P(y$ is odd$)$

$= \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$

3. There are 9 x 10 = 90 possible outcomes because the zero cannot be in the tens place. There are 9 x 2 = 18 favorable outcomes because numbers ending in 0 or 5 are divisible by 5.

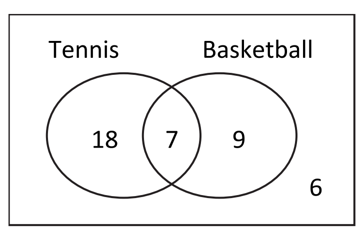
So, the probability is 18/90 = 1/5.

4. $\frac{2}{5} = \frac{y}{x + 2x + y}$ gives $y = 2x$.

$P(\text{red}) = \frac{x}{x + 2x + y} = \frac{x}{x + 2x + 2x} = \frac{x}{5x} = \frac{1}{5}$

LESSON 150

1. x = number of members who play basketball only
$2x$ = number of members who play tennis only
tennis only + both + basketball only + neither = 40
$2x + 7 + x + 6 = 40$; $x = 9$



2. 18 students

3. P(not tennis) = 1 – P(tennis) = 1 – (18 + 7)/40 = 3/8

4. P(tennis or basketball) = (18 + 7 + 9)/40 = 17/20

LESSON 152

1.

	Juice	Soda	Total
Juniors	12	13	25
Seniors	9	16	25
Total	21	29	50

2. P(juice) = 21/50
P(juice|junior) = (juice and junior)/junior = 12/25
The two events are not independent because P(juice|junior) ≠ P(juice).
You could compare P(junior) and P(junior|juice).
P(junior) = 25/50 = 1/2
P(junior|juice) = (junior and juice)/juice = 12/21 = 4/7

LESSON 154

1.

X	−20	20	12
$P(Y)$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

2. $E(X)$ = (−20)(3/8) + 20(1/2) + 12(1/8) = 4
The game is favorable to the player because the expected value is positive. There is no right or wrong answer for whether to play the game as long as you can give reasoning for your answer.

LESSON 157

1. possible outcomes = $P(9, 3)$
favorable outcomes = permutations of choosing 3 boys out of 5 = $P(5, 3)$
probability = $\frac{P(5,3)}{P(9,3)} = \frac{60}{504} = \frac{5}{42}$

2. possible outcomes = $P(9, 3)$
favorable outcomes = ways of choosing 1 girl out of 4 × permutations of choosing 2 boys out of 5 = 4 × $P(5, 2)$
probability = $\frac{4 \times P(5,2)}{P(9,3)} = \frac{4 \times 20}{504} = \frac{10}{63}$

3. possible outcomes = $P(9, 3)$
favorable outcomes = all outcomes – permutations of choosing 3 girls out of 4 = $P(9, 3) - P(4, 3)$
probability = $\frac{P(9,3) - P(4,3)}{P(9,3)} = \frac{504 - 24}{504} = \frac{20}{21}$
You could use the complement rule.
P(at least one position is filled by a boy)
= 1 – P(all three positions are filled by girls)
$= 1 - \frac{P(4,3)}{P(9,3)} = 1 - \frac{24}{504} = \frac{20}{21}$

LESSON 159

1. Let x be each angle of the quadrilateral. Angles in a quadrilateral add up to 360°, so $4x = 360$ and $x = 90$. This means that each angle must be a right angle. A quadrilateral with four right angles is a rectangle.

2. A regular polygon with n sides has n angles.
one interior angle = interior angle sum/# of angles
= $180(n-2)/n$

3. exterior angle sum of any polygon = 360°
one exterior angle = exterior angle sum/ # of angles
= $360/n$

LESSON 160

1. A, B, E

LESSON 161

1. A polygon with n sides can be divided into $n - 2$ triangles by the diagonals drawn from one vertex. Because the interior angle sum of each triangle is 180°, the interior angle sum of the polygon is $180(n-2)$°.

LESSON 162

1. interior angle sum = 180(5 – 2) = 540°
one interior angle = 540/5 = 108°
$\triangle EAD$ is an isosceles △ with vertex angle 108°.
$108 + 2a = 180$
$a = 36$
$b = m\angle D - a = 108 - 36 = 72$

2. 3. Base angles of an isosceles triangle are congruent.
5. SAS
6. CPCTC
7. Definition of isosceles triangle

LESSON 163

1. Statements (Reasons)
   1. $m\angle C = 90°$ (Given)
   2. $m\angle A + m\angle B + m\angle C = 180°$ (Angles in a triangle add up to 180°. See the Triangle Sum Theorem [32.1].)
   3. $m\angle A + m\angle B = 180 - m\angle C$ (Subtraction Property)
   4. $m\angle A + m\angle B = 180 - 90$ (Substitution Property)
   5. $m\angle A + m\angle B = 90$ (Simplify)
   6. $m\angle A < 90$, $m\angle B < 90$ (Definition of less than)
   7. $m\angle A < m\angle C$, $m\angle B < m\angle C$ (Substitution Property)
   8. $BC < AB$, $AC < AB$ (The larger angle has the longer opposite side. See Theorem 52.2.)

LESSON 164

1. Both pairs of opposite sides are parallel.
Both pairs of opposite sides are congruent.
One pair of opposite sides is parallel and congruent.

LESSON 165

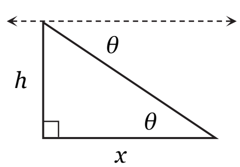
1. There are two pairs of similar triangles, so set up two proportions. Let $EF = x$ and $BF = y$. Then $DF = 48 - y$.

$\triangle BEF \sim \triangle BCD$	$\triangle DEF \sim \triangle DAB$
$\frac{EF}{CD} = \frac{BF}{BD}$	$\frac{EF}{AB} = \frac{DF}{DB}$
$\frac{x}{24} = \frac{y}{48}$	$\frac{x}{12} = \frac{48 - y}{48}$
$48x = 24y$	$48x = 12(48 - y)$
$y = 2x$	$4x = 48 - y$
	$4x = 48 - 2x$
	$x = 8$

So, the intersection is 8 m above the ground.

LESSON 166

1. $h = 50000 - 10000$
$= 40000$ feet
$x = 10$ miles x 5280 feet
$= 52800$ feet
$\theta = \tan^{-1}(h/x) \approx 37°$
The angle of depression is about 37 degrees.



LESSON 167

1. $m\widehat{BC} = 180 - m\widehat{CD} = 180 - 60 = 120°$
$m\widehat{AE} = 180 - m\widehat{AB} - m\widehat{DE} = 180 - 46 - 46 = 88°$

$m\angle 1 = 90°$ (inscribed in a semicircle)
$m\angle 2 = (m\widehat{AB} + m\widehat{DE})/2 = 46°$ (chord-chord angle)
$m\angle 3 = 90°$ (inscribed in a semicircle)
$m\angle 4 = m\widehat{CD}/2 = 30°$ (inscribed angle)
$m\angle 5 = m\widehat{BC}/2 = 60°$ (inscribed angle)
$m\angle 6 = m\widehat{CD}/2 = 30°$ (chord-tangent angle)
$m\angle 7 = (m\widehat{BAD} - m\widehat{CD})/2 = 60°$ (secant-tangent angle)

There are many ways to find these angle measures that are all correct. For example, you could use the Triangle Exterior Angle Theorem [32.2] to find $m\angle 7 = m\angle 3 - m\angle 6 = 90 - 30 = 60°$.

LESSON 168

1. A circle has 360°. There are 12 hours on a clock, so each number position is 360/12 = 30° around the clock. At 2:00, the minute hand is on the 12 and the hour hand is on the 2. So, the angle between them is 2(30) = 60°.

LESSON 169

1. sphere SA = sphere V

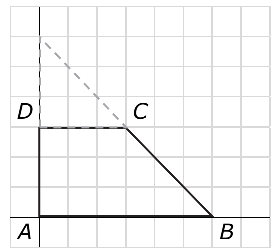
$4\pi r^2 = \frac{4}{3}\pi r^3$

$3(4)\pi r^2 = 4\pi r^3$

$r = 3$

2. All cross sections are rectangles. The largest possible cross section is a rectangle with width = diameter = 10 and height = 6. So, its area is 10(6) = 60 cm^2.

3.



The solid is a truncated cone, a cone whose top is cut off. The entire cone has radius 6 and height 6. The cut-off cone has radius 3 and height 3.

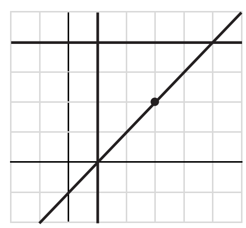
V = entire cone V – cut-off cone V

$= \frac{1}{3}\pi(6)^2(6) - \frac{1}{3}\pi(3)^2(3) = 63\pi$

4. rate = 12 – 3 = 9 ft^2/min
volume = 10(10)(10) = 1000 ft^2
time = volume / rate = 1000/9 ≈ 111
So, it will take about 111 minutes.

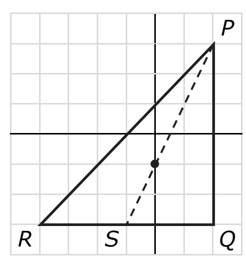
LESSON 170

1.



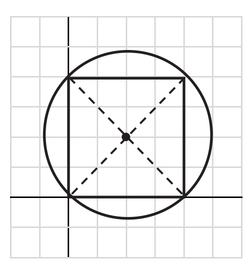
The circumcenter of a right triangle is the midpoint of the hypotenuse, so the circumcenter is at (3, 2).

2.



A centroid divides a median in the ratio 2:1.
Th point (0, −1) divides median $\overline{PS}$ in the ratio 2:1, so the centroid is at (0, −1).

3.



The circumcircle has center (2, 2) and radius $2\sqrt{2}$, so the standard equation is $(x - 2)^2 + (y - 2)^2 = 8$.

LESSON 172

1. Emma and Brian must sit next to each other, so group them together and treat them as one person.

possible outcomes = $P(8, 8)$

favorable outcomes = permutations of 7 people with Emma-Brian + permutations of 7 people with Brian-Emma = $P(7, 7) + P(7, 7) = P(7, 7) \times 2$

probability = $\frac{P(7, 7) \times 2}{P(8, 8)} = \frac{7! \times 2}{8!} = \frac{1}{4}$

2. The vowels should be placed second and fourth.

possible outcomes = $P(5, 5)$

favorable outcomes = permutations of 3 consonants with A second and E fourth + permutations of 3 consonants with E second and A fourth = $P(3, 3) \times 2$

probability = $\frac{P(3, 3) \times 2}{P(5, 5)} = \frac{3! \times 2}{5!} = \frac{1}{10}$

You could use the counting principle.

possible outcomes = 5 × 4 × 3 × 2 × 1 = 120

favorable outcomes = 3 × 2 × 2 × 1 × 1 = 12

1st place: 3 consonants

2nd place: 2 vowels

3rd place: 3 – 1 = 2 consonants

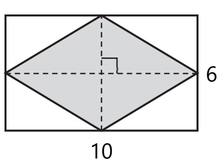
4th place: 2 – 1 = 1 vowel

5th place: 3 – 2 = 1 consonant

probability = 12/120 = 1/10

LESSON 174

1.



The area of the rhombus is half the area of the rectangle, so the probability is 1/2.