LESSON 109 Applications of Logarithms

☐ REFRESH YOUR SKILLS

(Lesson 97) The function $y = 3000(0.97)^t$ models the population of a town after t years.

1. What is the initial population of the town?

2. By what factor does the population decrease each year?

3. By what percent does the population decrease each year?

(Lesson 98) Adam put \$2,000 in an account that earns 4% interest compounded annually.

4. Write an exponential function to model the balance of the account, y, after t years.

5. What will be the balance after 5 years? Round to the nearest dollar.

(Lesson 106) Solve using logarithms. Round to four decimal places.

6. $7^x = 3$

7. $5 \cdot e^x = 25$

☐ SOLVING WORD PROBLEMS INVOLVING LOGARITHMS

Recall that many real-life situations involving exponential growth and decay can be modeled using an exponential function of the form $y = ab^x$. When the unknown is in an exponent, you can use logarithms to solve exponential equations.

➔ **EXAMPLE** The function $y = 200(1.03)^t$ models the balance of an account t years after the account was opened. How long will it take the balance to double?

As calculated on the right, it will take about 23 years for the balance to double.

$400 = 200(1.03)^t$	Plug in y = 400.
$(1.03)^t = 2$	Isolate the exponential.
$t = \log_{1.03} 2$	Rewrite in logarithmic form.
$t = \ln 2 / \ln 1.03$	Change-of-base formula
$t \approx 23$ years	Use a calculator.

➔ **TRY IT 8.** The function $y = 50(1.05)^t$ models the bear population in a region after t years. How long will it take the bear population to double?

Sometimes you may need to first write an exponential function that models the situation.

➔ **EXAMPLE** The value of a car is \$20,000. It loses 20% of its value every year. a) Write an exponential function to model the value of the car, y, after t years. b) How long will it take the car to be worth half its purchase price?

a. a = initial value = 20000
b = decay factor = 100% − 20% = 0.8
The function is $y = 20000(0.8)^t$.

b. When y = 10000, t = 3.10628....
It will take about 3 years.

➔ **TRY IT 9.** The population of a city is $50{,}000$ and is decreasing at a rate of 5% per year.

a. Write an exponential function to model the population, y, after t years.

b. How long will it take the population to drop to $30{,}000$?

☐ **EXERCISE YOUR SKILLS**

Solve. Round to the nearest whole number.

10. The function $y = 200(0.75)^t$ models the amount, in milligrams, of a certain drug remaining in the body after t hours.

a. What was the initial amount taken?

b. How much of the drug will remain in the body after 6 hours?

c. How long will it take for half the initial amount to leave the body?

d. How long will it take for only 10 mg to remain in the body?

11. A certain radioactive substance decays exponentially. The function $y = 100e^{-t/20}$ models the mass, in grams, of the substance remaining after t years.

a. What was the initial amount present?

b. How much of the substance will remain after 30 years?

c. How long will it take for the substance to decay to 10 grams?

d. How long will it take for the substance to decay to half the initial amount?

12. $\$5000$ is deposited in an account that earns 3% interest compounded annually.

a. Write an exponential function to model the balance, y, after t years.

b. What will be the balance after 10 years?

c. How long will it take for the balance to triple?

13. A cup of coffee contains 100 mg of caffeine. Caffeine leaves a body at 17% per hour.

a. Write an exponential function to model the amount, y, of caffeine in the body t hours after drinking a cup of coffee.

b. How much caffeine will be in the body 6 hours after drinking a cup of coffee?

c. How long will it take for half the caffeine to leave the body after drinking a cup of coffee?

14. A culture of 100 bacteria doubles every three hours.

a. Write an exponential function to model the number of bacteria, y, after t hours.

b. How many bacteria will be there after 12 hours?

c. How long will it take for the number of bacteria to reach $100{,}000$?