LESSON 129 Graphing Rational Functions

☐ REFRESH YOUR SKILLS

(Lesson 10) Find the intercepts.

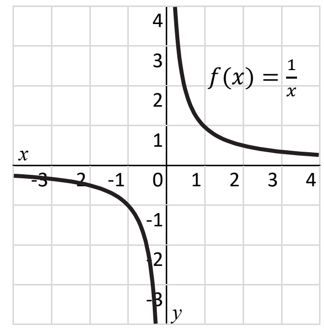
1. $y = 2x + 1$

(Lessons 94 & 107) Find the asymptote.

2. $f(x) = 2^x$

3. $f(x) = \log_2 x$

☐ FINDING ASYMPTOTES OF LINEAR RATIONAL FUNCTIONS



A **rational function** is a function that can be written as the quotient of two polynomials. The graphs of rational functions have various shapes depending on their polynomials.

A **linear rational function** is a rational function whose numerator is a number or a polynomial of degree 1 and whose denominator is a polynomial of degree 1. The graphs of linear rational functions are called **hyperbolas**, which consist of two separate curves opening in opposite directions. Shown on the right is the simplest hyperbola, the graph of the reciprocal function $f(x) = \frac{1}{x}$.

Notice that this graph has a vertical asymptote at the y-axis or $x = 0$ because the function is undefined when x is 0. It also has a horizontal asymptote at the x-axis or $y = 0$ because the function approaches 0 when x gets larger and larger (or smaller and smaller).

In general, a linear rational function in the form $f(x) = \frac{ax+b}{cx+d}$ has two asymptotes shown on the right, one vertical and one horizontal. The vertical asymptote is simply the value of x that makes $cx + d$ zero and thus makes $f(x)$ undefined.

Asymptotes of $f(x) = \dfrac{ax+b}{cx+d}$

$x = -\dfrac{d}{c}$ and $y = \dfrac{a}{c}$

What about the horizontal asymptote? Imagine that x gets larger and larger (or smaller and smaller). Then $ax + b$ will get closer to ax, $cx + d$ will get closer to cx, and $f(x)$ will get closer to $ax/cx = a/c$. This is the horizontal asymptote.

➔ **EXAMPLE** Find the vertical asymptote and horizontal asymptote of $f(x) = \frac{2x+1}{x+1}$.

a. $x + 1 = 0$
$x = -1$

To find the vertical asymptote, set denominator = 0 and solve for x.

b. $y = \dfrac{2}{1} = 2$

To find the horizontal asymptote, divide the coefficients of x.

➔ **TRY IT** Find the vertical asymptote and horizontal asymptote.

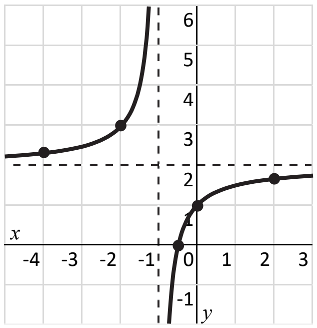
4. $f(x) = \dfrac{4}{x-3}$

5. $f(x) = \dfrac{3x-2}{x+5}$

☐ GRAPHING LINEAR RATIONAL FUNCTIONS

Like any function, linear rational functions can be graphed by plotting points. Draw the asymptotes using dashed lines, plot points including the intercepts, then draw a hyperbola through the points so that it approaches but never touches the asymptotes.

➔ EXAMPLE Graph $f(x) = \frac{2x+1}{x+1}$.



1. Draw the asymptotes $x = -1$ and $y = 2$.
2. Plot points including the intercepts.
   (0, 1), (−1/2, 0), (2, 1.66), (−2, 3), (−4, 2.33), …
3. Draw a hyperbola through the points.

➔ TRY IT Graph.

6. $f(x) = \dfrac{2}{x}$

7. $f(x) = \dfrac{x-1}{x+1}$

☐ IDENTIFYING FEATURES OF LINEAR RATIONAL FUNCTIONS

Now you can identify key features of linear rational functions from their graphs.

➔ EXAMPLE Identify the asymptotes, domain, and range of $f(x) = \frac{2x+1}{x+1}$.

As found in the first example, the vertical asymptote is $x = -1$ and the horizontal asymptote is $y = 2$. From the graph above, the domain is all real numbers except −1. The range is all real numbers except 2.

Notice that the domain is all real numbers except the vertical asymptote. The range is all real numbers except the horizontal asymptote.

➔ TRY IT Use your graphs to identify the asymptotes, domain, and range.

8. $f(x) = \dfrac{2}{x}$

9. $f(x) = \dfrac{x-1}{x+1}$

☐ EXERCISE YOUR SKILLS

Find the asymptotes.

10. $f(x) = \dfrac{3}{x-2}$

11. $f(x) = \dfrac{5x}{x+4}$

12. $f(x) = \dfrac{x+2}{x+7}$

13. $f(x) = \dfrac{4x-1}{2x-6}$

Graph. State the asymptotes, domain, and range.

14. $f(x) = \dfrac{3}{x}$

15. $f(x) = \dfrac{x}{x+1}$

16. $f(x) = \dfrac{x+1}{x}$

17. $f(x) = \dfrac{2x-1}{x-1}$