LESSON 130 Transforming Rational Functions

☐ **REFRESH YOUR SKILLS**

(Lesson 67) Describe the transformations from $y = x^2$.

1. $f(x) = -x^2 + 3$
2. $f(x) = 2(x-1)^2$

(Lesson 129) Find the asymptotes.

3. $f(x) = 1/x$

☐ **DESCRIBING TRANSFORMATIONS OF LINEAR RATIONAL FUNCTIONS**

Recall that all functions in the same family can be graphed by transforming the graph of their parent function. The parent function of linear rational functions is $y = \frac{1}{x}$.

Any linear rational function can be written in the standard form $f(x) = \frac{a}{x-h} + k$, where $a \neq 0$. When a linear rational function is in standard form, it is easy to graph using transformations because the values of a, h, and k tell us what transformations are taking place. By now you should be familiar with these values.

Parent function $y = \frac{1}{x}$

$$f(x) = \frac{a}{x-h} + k$$

Stretch or shrink vertically by $|a|$.
Reflect over the x-axis if $a < 0$.
Shift right or left h units.
Shift up or down k units.

➔ **EXAMPLE** Describe the transformations from $y = \frac{1}{x}$ to $f(x) = -\frac{3}{x+1} - 2$.

$y = \frac{1}{x}$ Parent function

$y = \frac{-3}{x}$ Scale vertically by 3. Reflect over the x-axis.

$f(x) = -\frac{3}{x+1} - 2$ Shift left 1 unit. Shift down 2 units.

➔ **TRY IT** Describe the transformations from $y = \frac{1}{x}$ to $f(x)$.

4. $f(x) = \frac{4}{x+3}$
5. $f(x) = -\frac{1}{x-2} + 5$

☐ **FINDING ASYMPTOTES OF LINEAR RATIONAL FUNCTIONS**

Notice that, when a linear rational function is in standard form, it is also easy to find the asymptotes as well because the values of h and k tell us where the asymptotes are.

Shift the vertical asymptote, $x = 0$, of the parent function left or right h units, and you get $x = h$. Shift the horizontal asymptote, $y = 0$, of the parent function up or down k units, and you get $y = k$.

Asymptotes of $f(x) = \frac{a}{x-h} + k$

$x = h$ and $y = k$

➔ **EXAMPLE** Find the asymptotes of $f(x) = -\frac{1}{x+1} + 2$.

$x = h = -1$ Vertical asymptote

$y = k = 2$ Horizontal asymptote

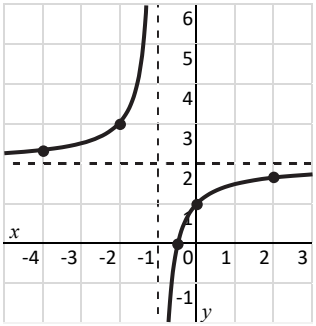
➔ **TRY IT** Find the asymptotes.

6. $f(x) = \frac{1}{x-2} + 5$

☐ GRAPHING LINEAR RATIONAL FUNCTIONS USING TRANSFORMATIONS

Now you can graph any linear rational function in standard form using transformations. The example below shows the final graph for readability after all the transformations are applied. Note that your graphs do not have to be exact. Just be sure to show their key features clearly.

➔ **EXAMPLE** Graph $f(x) = -\frac{1}{x+1} + 2$.



1. Graph $y = 1/x$.
2. Reflect over the x-axis to get $y = -1/x$
3. Shift left 1 unit and up 2 units to get $f(x)$.
4. Draw the asymptotes $x = -1$ and $y = 2$.

➔ **TRY IT** Graph.

7. $f(x) = \dfrac{2}{x-1}$

8. $f(x) = -\dfrac{1}{x} - 1$

What are the domain and range of the function above? Simply exclude the vertical asymptote from the domain and the horizontal asymptote from the range. So the domain of the function above is all real numbers except -1. The range is all real numbers except 2.

☐ EXERCISE YOUR SKILLS

Describe the transformations from $y = \frac{1}{x}$ to $f(x)$.

9. $f(x) = \dfrac{1}{x-3}$

10. $f(x) = \dfrac{1}{x+4} - 3$

11. $f(x) = -\dfrac{2}{x+5}$

12. $f(x) = \dfrac{3}{x-2} + 7$

Graph using transformations. State the asymptotes, domain, and range.

13. $f(x) = \dfrac{2}{x}$

14. $f(x) = -\dfrac{1}{x-2}$

15. $f(x) = \dfrac{3}{x+1}$

16. $f(x) = -\dfrac{1}{x-1} + 1$

(CHALLENGE) Write a linear rational function $f(x) = \frac{a}{x-h} + k$ given asymptotes and a point.

17. The vertical asymptote is $x = 3$.
The horizontal asymptote is $y = 2$.
The graph passes through $(2, 0)$.

18. The vertical asymptote is $x = -2$.
The horizontal asymptote is $y = 1$.
The graph passes through $(1, -1)$.