LESSON 132 Applications of Rationals

☐ **IDENTIFYING TYPES OF VARIATIONS** ···

As seen in the previous lesson, rational equations can be used to model many real-life situations. Equations representing direct and inverse variation are also examples of rational equations.

Direct variation is a relationship between two variables x and y that can be written in the form $y = kx$, or $k = y/x$, where k is a nonzero constant. Direct variation $y = kx$ means that y is directly proportional to x, or y varies directly with x. The constant k is called the **constant of variation**.

> Direct variation
>
> $$y = kx \text{ or } k = \frac{y}{x}$$

Inverse variation is a relationship between two variables x and y that can be written in the form $y = k/x$, or $k = xy$, where k is a nonzero constant. Inverse variation $y = k/x$ means that y varies inversely with x. The constant k is also called the constant of variation.

> Inverse variation
>
> $$y = \frac{k}{x} \text{ or } k = xy$$

To identify the type of variation in an equation, simply solve for y.

➔ **TRY IT** Identify as direct variation, inverse variation, or neither.

a. $x + y = 1$

$y = -x + 1$

Neither

b. $2y = 4x$

$y = 2x$

Direct variation with k = 2

c. $2xy = 6$

$y = 3/x$

Inverse variation with k = 3

➔ **TRY IT** Identify as direct variation, inverse variation, or neither.

1. $3y = 9x$

2. $x - y = 2$

3. $xy - 3 = 2$

☐ **SOLVING WORD PROBLEMS INVOLVING VARIATION** ···

To solve a word problem involving direct variation, 1) define two variables x and y, 2) plug the given values into $y = kx$ and find the value of k, 3) use k to write an equation that models the given situation, and 4) use the equation to find the unknown.

➔ **EXAMPLE** The distance a car travels varies directly with the amount of gas it uses. A car uses **7** gallons of gas to travel **210** miles. How much gas will the car use to travel **180** miles?

> The phrase "varies directly with" indicates direct variation.

1. Let x = amount of gas used
   Let y = distance traveled

2. Plug x = 7 and y = 210 into $y = kx$.
   210 = 7k, so k = 30.

3. The equation is $y = 30x$.

4. When y = 180, x = 6.
   The car will use 6 gallons of gas.

➔ **TRY IT 4.** The distance sound travels varies directly with the time it travels. Sound travels **1,700** meters in **5** seconds in air. How far will it travel in **12** seconds?

Solving an inverse variation problem is just like solving a direct variation problem except that you use $y = k/x$, or $k = xy$, instead of $y = kx$.

➜ **EXAMPLE** The resale value of a car varies inversely with its age. If a 2-year-old car costs $18,000, what will be the value of the car when the car is 5 years old?

> The phrase "varies inversely with" indicates inverse variation.

1. Let x = age in years of the car
   Let y = resale value of the car

2. Plug x = 2 and y = 18000 into $y = k/x$.
   18000 = $k/2$, so k = 36000.

3. The equation is $y = 36000/x$.

4. When x = 5, y = 7200.
   The value of the car will be $7,200.

➜ **TRY IT 5.** The time it takes to finish a painting job varies inversely with the number of people working on the job. If it takes 3 hours for 4 people to finish a painting job, how long will it take for 6 people to finish the job?

☐ **EXERCISE YOUR SKILLS** ···

Identify as direct variation, inverse variation, or neither.

6. $xy = -2$ **7.** $3y = -6$ **8.** $5xy - 1 = 0$ **9.** $x - 3y = 0$

Write a variation equation for the situation, then solve.

10. Suppose y varies directly with x, and $y = 15$ when $x = 3$. Find x when $y = 30$.

11. Suppose y varies inversely with x, and $y = 4$ when $x = -5$. Find y when $x = 2$.

12. The distance a car travels varies directly with the time it travels. If a car travels 72 kilometers in 3/4 hour, how far will it travel in 4 hours?

13. The height of an object varies directly with the length of its shadow. If a tree 12 feet tall casts a shadow 18 feet long, how long will be the shadow of a tree that is 16 feet tall?

14. The time it takes to empty a water tank varies inversely with the rate of pumping. If a pump can empty a tank in 36 minutes at 12 gallons per minute, how long will it take to empty the same tank at 16 gallons per minute?

15. The current in an electrical circuit varies inversely with the resistance. If the current is 9 amperes when the resistance is 4 ohms, what will be the current when the resistance is 12 ohms?

16. The weight of an object on Earth is directly proportional to, or varies directly with, its weight on Mars. An object that weighs 50 pounds on Earth would weigh about 19 pounds on Mars. How much will an 80-pound object on Earth weigh on Mars?