

## LESSON 140

1.  $6\sqrt{2}$     2. (1, 2)    3. D    4.  $2\sqrt{10}$   
 5. 4    6. B, D    7.  $1/4$   
 8.  $(0, -1), (3, 2)$     9.  $(1, -2)$   
 10.  $k = 2$

Worked-out solutions:

1.  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$   
 $= \sqrt{(4 - (-2))^2 + (9 - 3)^2} = \sqrt{72} = 6\sqrt{2}$

2.  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 - 2}{2}, \frac{-3 + 7}{2}\right) = (1, 2)$

4.  $(h, k) = \text{midpoint between } (2, -1) \text{ and } (-4, -3)$   
 $= \left(\frac{2 - 4}{2}, \frac{-1 - 3}{2}\right) = (-1, -2)$

$r = \text{distance between } (-1, -2) \text{ and } (2, -1)$   
 $= \sqrt{(2 - (-1))^2 + (-1 - (-2))^2} = \sqrt{10}$

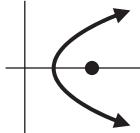
$hkr = (-1)(-2)(\sqrt{10}) = 2\sqrt{10}$

5. Convert to standard form by completing the square.

$$\begin{aligned} x^2 + y^2 - 6x + 4y - 3 &= 0 \\ x^2 + y^2 - 6x + 4y &= 3 \\ x^2 - 6x + y^2 + 4y &= 3 \\ (x^2 - 6x + 9) + (y^2 + 4y + 4) &= 3 + 9 + 4 \\ (x - 3)^2 + (y + 2)^2 &= 16 = 4^2 \end{aligned}$$

6. A) The focus is right of the directrix, so the parabola opens right.  
 B) The vertex, the midpoint between the focus and the directrix, is  $(2, 0)$ .  
 C) The axis of symmetry, the line passing through the focus and the vertex, is  $y = 0$ .  
 D)  $(4, 4)$  is equidistant from the focus and the directrix, so it is on the parabola.

7. Let  $(x, y)$  be a point on the parabola.



distance to the directrix = distance to the focus

$$\begin{aligned} \sqrt{(x - x)^2 + (y + 1)^2} &= \sqrt{(x - 0)^2 + (y - 1)^2} \\ \sqrt{(y + 1)^2} &= \sqrt{x^2 + (y - 1)^2} \quad \text{Simplify.} \\ (y + 1)^2 &= x^2 + (y - 1)^2 \quad \text{Square both sides.} \\ y^2 + 2y + 1 &= x^2 + y^2 - 2y + 1 \quad \text{Expand.} \\ y = \frac{1}{4}x^2 & \quad \text{Solve for } y. \end{aligned}$$

8.  $(x - 2)^2 + (x - 1)^2 = 5$     Plug eq2 into eq1.  
 $2x^2 - 6x = 0$     Write in standard form.  
 $2x(x - 3) = 0$     Solve for  $x$ .  
 $x = 0, x = 3$   
 $y = 0 - 1 = -1$     Find  $y$  when  $x = 0$ .  
 $y = 3 - 1 = 2$     Find  $y$  when  $x = 3$ .  
 Solutions:  $(0, -1), (3, 2)$

9.  $y = x - 3$     Solve eq2 for  $y$ .  
 $x - 3 = x^2 - x - 2$     Plug eq2 into eq1.  
 $x^2 - 2x + 1 = 0$     Write in standard form.  
 $(x - 1)^2 = 0$     Solve for  $x$ .  
 $x = 1$   
 $y = 1 - 3 = -2$     Find  $y$  when  $x = 1$ .  
 Solution:  $(1, -2)$

10. The graphs intersect at one point.  
 = The system has one real solution.

$$\begin{aligned} 2x + 3 &= (x + 1)^2 + k \quad \text{Plug eq2 into eq1.} \\ x^2 + k - 2 &= 0 \end{aligned}$$

The discriminant,  $b^2 - 4ac$ , must be zero.

$$\begin{aligned} 0^2 - 4(1)(k - 2) &= 0 \quad \text{Set discriminant = 0.} \\ k = 2 & \quad \text{Solve for } k. \end{aligned}$$