LESSON 147 Applications of Sequences and Series

☐ SOLVING WORD PROBLEMS INVOLVING SEQUENCES

When a word problem involving a sequence asks for a term, write out the first few terms to determine the type of the sequence. Then write and use the explicit rule to find the term asked for.

S**imple interest** is calculated only on the principal. For example, if you invest \$100 at a simple interest rate of 5% per year, you will earn $\$100 \times 5\% = \$100 \times 0.05 = \$5$ interest each year.

➔ **EXAMPLE** Eli put \$3,000 in her savings account with a simple interest rate of 3% per year. What will be the balance of her account at the end of the 9th year?

Note that, at the end of the 1st year, the balance will be \$3000 + 3% interest on \$3000. So a_1 = 3090, not 3000.

1. 3090, 3180, 3270, 3360, 3450, ...
   This is an arithmetic sequence.
2. a_1 = 3090 and d = 3000 × 3% = 90, so a_n = 90n + 3000.
3. When n = 9, a_9 = 3810.
   The balance will be \$3,810.

➔ **TRY IT 1.** Samantha put \$5,000 in her savings account with a simple interest rate of 4% per year. What will be the balance of her account at the end of the 10th year?

➔ **EXAMPLE** A ball is dropped from a height of 256 feet. After the ball hits the floor, it rebounds to 50% of its previous height. How high will the ball rebound after its 9th bounce?

Note that, in our sequence, we define a_n as the height AFTER the nth bounce. So a_1 = 128, not 256.

1. 128, 64, 32, 16, ...
   This is a geometric sequence.
2. a_1 = 128 and r = 50% = 0.5, so a_n = 128(0.5)$^{n-1}$.
3. When n = 9, a_6 = 0.5.
   The ball will bounce to 0.5 feet.

➔ **TRY IT 2.** A ball is dropped from a height of 128 feet. After the ball hits the floor, it rebounds to 75% of its previous height. How high will the ball rebound after its seventh bounce?

☐ SOLVING WORD PROBLEMS INVOLVING SERIES

When a word problem involving a sequence asks for a sum or total, determine the type of the sequence. Then use the appropriate series formula to find the sum asked for.

➔ **EXAMPLE** A theater has 20 rows of seats. The first row has 40 seats, the second row has 46 seats, the third row has 52 seats, and so on. How many seats are there in all?

The key phrase "in all" indicates a series.

1. This is an arithmetic sequence.
2. a_1 = 40 and d = 6, so a_n = 6n + 34.
3. Find the sum of the first 20 terms.
   S_{20} = (20/2)(a_1 + a_{20})
   = 10(40 + 154) = 1940
   There are 1,940 seats in all.

➔ **TRY IT 3.** A concert hall has 24 rows of seats. There are 20 seats in the first row, 23 seats in the second row, 26 seats in the third row, and so on. How many seats are there in all?

➔ **EXAMPLE** The starting salary for Lucas's job is $52,000 a year. He is to receive a 5% raise each year. What is the total amount that Lucas earns in his first 5 years?

Be careful that r is 1.05, not 0.05, because
new salary = previous salary + previous salary × 0.05
= previous salary × (1 + 0.05)
= previous salary × (1.05)

1. This is a geometric sequence.
2. a_1 = 52000 and r = 1.05.
3. Find the sum of the first 5 terms.
   $S_5 = a_1(1 - r^5)/(1 - r)$
   $= 52000(1 - 1.05^5)/(1 - 1.05)$
   $= 287332.825$
   Lucas earns about $287,333 in total.

➔ **TRY IT 4.** Mark started a job that paid $62,000 a year. Each year his salary was increased by 4% over the previous year's salary. What is the total amount that Mark earned in 6 years?

☐ **EXERCISE YOUR SKILLS**

Solve. Round to the nearest whole number, if necessary.

5. Henry put $1,000 in his savings account with a simple interest rate of 5% per year. What will be the balance of his account at the end of the 15th year?

6. A tree in Molly's backyard grows 1.2 feet per year. It was 5 feet tall at the beginning of this year. What will be the height of the tree at the beginning of the 8th year?

7. Theodore is studying algebra. He scored 72% on his first quiz. Then he scored 76% and 80% on his next two quizzes. If this pattern continues, what will be his score on his 7th quiz?

8. As a car gets older, its resale value goes down by 10% each year. Kris bought a new car for $20,000. What will be the value of the car after 6 years? (*Hint*: What is r? 0.1 or 0.9?)

9. A new website had a total of 500 hits this week. The traffic is estimated to increase by 100% per week. How many hits will the website have in the 5th week? (*Hint*: What is r? 1 or 2?)

10. During a science experiment, Jessica found that bacteria double every hour. There were 10 bacteria at 1 pm. How many bacteria will be there at 8 pm on the same day?

11. Eva worked for a company for 12 years. Her starting annual salary was $55,000. Each year she received a raise of $2500. What was the total amount that Eva earned with this company?

12. While training for a marathon, Lucas increases his distance by 0.5 miles each day. If he runs 2 miles on the first day, what will be that total distance that he runs for 30 days of training?

13. Shylo took medication for 8 days. On the first day, she took 300 mg. Then she took one half the amount taken on the previous day. What is the total amount of medication taken?

14. A new book sells 2,000 copies its first week, and its sales drop by 10% each week. What is the total number of copies sold in the first 10 weeks?