LESSON 167

1. The answer is A.

B) 1 is real but not irrational.

C) $\sqrt{2} + \left(-\sqrt{2}\right) = 0$ is rational.

D) $\left(\sqrt{2}\right)\left(\sqrt{2}\right) = 2$ is rational.

2. $i^3 + i^4 + i^5 + i^6$

$= i^2 \cdot i + (i^2)^2 + (i^2)^2 \cdot i + (i^2)^3$

$= (-1) \cdot i + (-1)^2 + (-1)^2 \cdot i + (-1)^3$

$= -i + 1 + i - 1$

$= 0$

3. $(3 - i)(2 + 5i) + 4i$

$= 6 + 15i - 2i - 5i^2 + 4i$

$= 6 + 15i - 2i - 5(-1) + 4i$

$= 6 + 15i - 2i + 5 + 4i$

$= 11 + 17i$

4. $(x - 1)(3x - 2) = 0$

$x - 1 = 0$ or $3x - 2 = 0$ — Zero-product property

$x = 1, x = 2/3$ — Solve for x.

5. $3x^2 + 8x - 3 = 0$

$(3x - 1)(x + 3) = 0$ — Factor the quadratic.

$3x - 1 = 0$ or $x + 3 = 0$ — Zero-product property

$x = 1/3\,, x = -3$ — Solve for x.

$k = -3$ because $k < 0$.

$3k + 1 = 3(-3) + 1 = -8$

6. $2x^2 + 6x + 5 = 0$

$x = \dfrac{-6 \pm \sqrt{-4}}{2(2)}$ — Quadratic formula

$x = \dfrac{-6 \pm 2i}{4}$ — Simplify.

$x = -\dfrac{3}{2} \pm \dfrac{1}{2}i$

$a + b = -\dfrac{3}{2} + \dfrac{1}{2} = -1$

7. The discriminant, $b^2 - 4ac$, must be negative.

$6^2 - 4(3)k < 0$ — Set discriminant < 0

$36 - 12k < 0$ — Solve for k.

$k > 3$

8. $(x - 4)(2x + 3) = x - 7$

$2x^2 - 5x - 12 = x - 7$ — Simplify each side.

$2x^2 - 6x - 5 = 0$ — Write in standard form.

Sum of roots $= -b/a = -(-6)/2 = 3$

9. Sum of roots $= -b/a = -b/3 = 3$

Product of roots $= c/a = c/3 = 2$

$b = -9, c = 6$

10. Let $x =$ first number

$7 - x =$ second number

Product $= 10$, so $x(7 - x) = 10$.

Solve for x, and you get $x = 2$ and $x = 5$.

The numbers are 2 and 5.

11. Let $x =$ length of the rectangle

$3x =$ width of the rectangle

Area = (width)(length) $= 60$, so $x(3x) = 60$.

Solve for x, and you get $x = 2\sqrt{5}$ and $x = -2\sqrt{5}$.

The length is $2\sqrt{5}$ cm. The width is $6\sqrt{5}$.

12. The answer is D.

The leading coefficient must be negative because the parabola opens down, so eliminate A and C.

$(3, 0)$ is on the graph, so choose D.

13. The answers are A, C, and D.

A) The parabola opens up because the leading coefficient is positive.

B) The x-intercepts are -5 and 1.

C) The axis of symmetry is halfway between the two x-intercepts. $x = (-5 + 1)/2 = -2$

D) The axis of symmetry is the x-coordinate of the vertex. The vertex is $(-2, f(-2)) = (-2, -9)$.

E) The y-intercept is $f(0) = -5$.

14. Convert to vertex form by completing the square.

$f(x) = x^2 + 6x - 1$

$= x^2 + 6x + 9 - 9 - 1$

$= (x + 3)^2 - 9 - 1$

$= (x + 3)^2 - 10$

The vertex is $(-3, -10)$.

15. $a < 0$ because the parabola opens down.

$c < 0$ because the y-intercept is negative.

$b > 0$ because the axis of symmetry $(-b/2a)$ is positive and $a < 0$.

16. The discriminant, $b^2 - 4ac$, must be zero.

$4^2 - 4(2)c = 0$ — Set discriminant $= 0$.

$c = 2$ — Solve for c.

17. The discriminant, $b^2 - 4ac$, must be negative.

$(-6)^2 - 4k(3) < 0$ — Set discriminant < 0.

$k > 3$ — Solve for k.

18. $f(x) = x^2$ Parent function

$y = -x^2$ Reflect over the x-axis.

$y = -(x-1)^2$ Shift right 1 unit.

$g(x) = -(x-1)^2 - 2$ Shift down 2 units.

$g(x) = -x^2 + 2x - 3$ Standard form

19. First, find the equation of the parabola.

$y = a(x-h)^2 + k$ Use vertex form.

$y = a(x-2)^2$ Plug in vertex $(2, 0)$.

$-1 = a(0-2)^2$ Plug in point $(0, -1)$.

$a = -1/4$ Solve for a.

$y = -\frac{1}{4}(x-2)^2$ Vertex form

$y = -\frac{1}{4}x^2 + x - 1$ Standard form

Second, determine the inequality sign.

The line is solid and $(0, -2)$ is a solution,

so the inequality is $y \leq -\frac{1}{4}x^2 + x - 1$.

20. $(x+2)(x-3) > 0$

The related equation has roots −2 and 3. Use them to create three intervals. Then test a point in each interval to determine the solution set.

$x < -2$	$-2 < x < 3$	$x > 3$
$x = -3$ is a solution.	$x = 0$ is not a solution.	$x = 4$ is a solution.

The solution set is $x < -2$ or $x > 3$.

21. $-16t^2 + 80t = 0$

$-16t(t-5) = 0$

$t = 0, t = 5$

The ball will hit the ground after 5 seconds.