LESSON 168

1. $(x^2+1)(x^4+x^2+1)$
$= x^6+x^4+x^2+x^4+x^2+1$
$= x^6+2x^4+2x^2+1$
$\neq x^6+1$
The equation is NOT an identity.

2. $2x^3+16$
$= 2(x^3+8)$ Factor out the GCF.
$= 2(x+2)(x^2-2x+4)$ Sum of cubes

3. x^4+3x^2-4
$= u^2+3u-4$ Let $u = x^2$.
$= (u-1)(u+4)$ Factor.
$= (x^2-1)(x^2+4)$ Substitute back.
$= (x+1)(x-1)(x^2+4)$ Difference of squares

4. $3x^3+x^2-12x-4$
$= x^2(3x+1)-4(3x+1)$ Factor by grouping.
$= (3x+1)(x^2-4)$
$= (3x+1)(x+2)(x-2)$ Difference of squares

5. $x^2(x-5)^3(x+5)=0$
$x=0$ or $x-5=0$ or $x+5=0$ Zero-product property
$x=0, x=5, x=-5$ Solve for x.
$x=0$ has multiplicity 2.
$x=5$ has multiplicity 3.
$x=-5$ has multiplicity 1.

6. $x^3-x^2=x$
$x^3-x^2-x=0$ Standard form
$x(x^2-x-1)=0$ Factored form
$x=0$ or $x^2-x-1=0$ Zero-product property
$x=0, x=\frac{1\pm\sqrt{5}}{2}$ Solve for x.

7. $x^3-2x^2+5x=0$
$x(x^2-2x+5)=0$ Factored form
$x=0$ or $x^2-2x+5=0$ Zero-product property
$x=0, x=\frac{2\pm\sqrt{-16}}{2}$ Solve for x.
$x=0, x=1\pm 2i$ Simplify.

8. $(x-3)(x-2i)(x+2i)=0$ Factored form
$(x-3)(x^2+4)=0$ Multiply out.
$x^3-3x^2+4x-12=0$ Standard form

9. Sum of $2\pm\sqrt{5}=4$
Product of $2\pm\sqrt{5}=-1$
Quadratic factor with roots $2\pm\sqrt{5}=x^2-4x-1$
$x(x^2-4x-1)=0$ Factored form
$x^3-4x^2-x=0$ Standard form

10. $x(x-2)(x-1)^2=0$ Factored form
$x(x-2)(x^2-2x+1)=0$ Multiply out.
$(x^2-2x)(x^2-2x+1)=0$
$x^4-4x^3+5x^2-2x=0$ Standard form

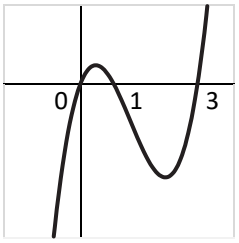
11. The answer is B.
The zeros are -3, -1, 2, and 3, so the function has factors $(x+3)$, $(x+1)$, $(x-2)$, and $(x-3)$.
x^2-9 is factored as $(x+3)(x-3)$.

12. The answer is D.
The zeros are 0 and 4, so eliminate A and C.
The leading coefficient is negative because the right end of the graph goes down, so choose D.

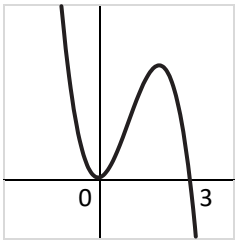
13. The answer is A.
The zeros are -1 and 2, so eliminate C and D.
Both zeros have an odd multiplicity because the graph crosses the x-axis at them, so choose A.

14. The answers are B and C.
A) The zero 1 has multiplicity 3.
B) The graph touches the x-axis at $x=-3$ because $x=-3$ has an even multiplicity of 2.
C) Test a point. $f(0)=9>0$
D) The leading term is $-x^5$. The degree is 5 (odd). The leading coefficient is -1 (negative). The left end goes up and the right end goes down.

15. $x^5+2x^4+x^3=0$ Set $f(x)=0$.
$x^3(x^2+2x+1)=0$ Factor out the GCF.
$x^3(x+1)^2=0$ Perfect square trinomial
$x=0$ (multiplicity 3, crosses the x-axis)
$x=-1$ (multiplicity 2, touches the x-axis)

16. 

The zeros are 0, 1, and 3.
Sketch the graph, or test a point in each interval created by the zeros.
The positive intervals are $(0,1)$ and $(3,\infty)$.

17. $f(x)=-x^2(x-3)$


The zeros are 0 (multiplicity 2) and 3 (multiplicity 1).
Sketch the graph, or test a point in each interval created by the zeros.
The positive intervals are $(-\infty,0)$ and $(0,3)$.

18. $f(x) = a(x+2)(x-2)(x-3)$ Factored form

$6 = a(0+2)(0-2)(0-3)$ Plug in $(0, 6)$.

$6 = 12a;\ a = 1/2$ Solve for a.

$f(x) = \frac{1}{2}(x+2)(x-2)(x-3)$ Plug in a.

$f(x) = \frac{1}{2}x^3 - \frac{3}{2}x^2 - 2x + 6$ Standard form

19. $f(x) = ax^2(x-2)^2$ Factored form

$3 = a(1^2)(1-2)^2$ Plug in $(1, 3)$.

$3 = a$ Solve for a.

$f(x) = 3x^2(x-2)^2$ Plug in a.

$f(x) = 3x^4 - 12x^3 + 12x^2$ Standard form

20. $x^3(x^2-5) + 4x = 0$

$x^5 - 5x^3 + 4x = 0$

$x(x^4 - 5x^2 + 4) = 0$

$x(x^2-1)(x^2-4)$

$x(x+1)(x-1)(x+2)(x-2) = 0$

$x = 0, x = -1, x = 1, x = -2, x = 2$

Sum $= 0 + (-1) + 1 + (-2) + 2 = 0$

21. $(2i)^3 + b(2i)^2 + c(2i) - 4 = 0$ Plug in $x = 2i$.

$8i^3 + 4bi^2 + 2ci - 4 = 0$ Simplify.

$-8i - 4b + 2ci - 4 = 0$

$(-4b - 4) + (2c - 8)i = 0$ Write as $a + bi$.

$-4b - 4 = 0$ and $2c - 8 = 0$

$b = -1, c = 4$