LESSON 170

1. $10^b = a$

2. $\log_2 1 = 0$ Zero rule
$\log_2 64 = \log_2 2^6 = 6$ Use $\log_b b^n = n$.
$\log_2 1 + \log_2 64 = 0 + 6 = 6$

3. $\log_7 7 = 1$ Identity rule
$\log_7 \frac{1}{49} = \log_2 7^{-2} = -2$ Use $\log_b b^n = n$.
$\log_7 7 + \log_7 \frac{1}{49} = 1 - 2 = -1$

4. The answer is D.

5. $\log_3 3^2 x^4$
$= \log_3 3^2 + \log_3 x^4$ Product rule
$= 2\log_3 3 + 4\log_3 x$ Power rule
$= 2 + 4\log_3 x$ Identity rule
$a + b = 2 + 4 = 6$

6. $\log_2 \frac{2^2 x^3}{5}$
$= \log_2 2^2 x^3 - \log_2 5$ Quotient rule
$= \log_2 2^2 + \log_2 x^3 - \log_2 5$ Product rule
$= 2\log_2 2 + 3\log_2 x - \log_2 5$ Power rule
$= 2 + 3\log_2 x - \log_2 5$ Identity rule

7. $\log_7 6x + 2\log_7 x - \log_7 3$
$= \log_7 6x + \log_7 x^2 - \log_7 3$ Power rule
$= \log_7 6x^3 - \log_7 3$ Product rule
$= \log_7 \frac{6x^3}{3}$ Quotient rule
$= \log_7 2x^3$ Simplify.

8. $\log(2x - 3) = \log(9 - x)$
$2x - 3 = 9 - x$ One-to-one property
$x = 4$ Solve for x.

9. $\ln x + \ln(x + 1) = \ln(4x - 2)$
$\ln x(x + 1) = \ln(4x - 2)$ Product rule
$x(x + 1) = 4x - 2$ One-to-one property
$x^2 - 3x + 2 = 0$ Standard form
$(x - 1)(x - 2) = 0$ Solve for x.
$x = 1, x = 2$

10. $\log_3 x + \log_3(x - 6) = 3$
$\log_3 x(x - 6) = 3$ Product rule
$x(x - 6) = 3^3$ Exponential form
$x^2 - 6x - 27 = 0$ Standard form
$(x + 3)(x - 9) = 0$ Solve for x.
$x = -3, x = 9$
$x = -3$ is extraneous, so the solution is $x = 9$.
$\log_3 a = \log_3 9 = \log_3 3^2 = 2$

11. $e^{x+2} - 1 = 6$
$e^{x+2} = 7$ Isolate the exponential.
$x + 2 = \ln 7$ Logarithmic form
$x = \ln 7 - 2$ Solve for x.

12. The answer is B.
$f(x)$ is $y = \log_2 x$ reflected over the y-axis.

13. The answer is A.
$(-2, 0)$ is on the graph, so eliminate B and D.
The graph does not involve a reflection, so choose A.

14. The answer is B.
$y = \ln x$ Parent function
$y = 2\ln x$ Stretch vertically by 2.
$y = 2\ln(-x)$ Reflect over the y-axis.
$f(x) = 2\ln(-x) - 3$ Shift down 3 units.

15. $f(x)$ is $y = \log x$ shifted down 4 units. Vertical shifts do not change the domain of a logarithmic function.
The domain of $y = \log x$ is $(0, \infty)$,
so the range of $f(x)$ is also $(0, \infty)$.

16. $f(x)$ is $y = \log x$ shifted right 1 unit and up 2 units.
The asymptote of the graph of $y = \log x$ is $x = 0$,
so the asymptote of the graph of $f(x)$ is $x = 1$.

17. The initial balance is \$2,000.
$2000(1.03)^t = 4000$
$(1.03)^t = 2$
$t = \log_{1.03} 2 = \frac{\ln 2}{\ln 1.03} = 23.44977 \ldots$
It will take about 23 years.

18. $100e^{-t/50} = 50$
$e^{-t/50} = 1/2$
$-t/50 = \ln(1/2)$
$t = -50\ln(1/2) = 34.65735 \ldots$
It will take about 35 years.

19. The initial population is $40{,}000$.

The decay factor is $100\% - 6\% = 94\% = 0.94$.

The function $y = 40000(0.94)^t$ models the population of the town after t years.

$40000(0.94)^t = 30000$
$(0.94)^t = 3/4$

$$t = \log_{0.94}(3/4) = \frac{\ln(3/4)}{\ln 0.94} = 4.64937\ \ldots$$

It will take about 5 years.

20. $\log_3 20 = \log_3(5 \cdot 2^2)$

$= \log_3 5 + \log_3 2^2$	Product rule
$= \log_3 5 + 2\log_3 2$	Power rule
$= m + 2n$	Substitute.