LESSON 173

1. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-7-(-1))^2 + (3-5)^2} = \sqrt{40} = 2\sqrt{10}$

2. $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right) = \left(\dfrac{5-3}{2}, \dfrac{1-9}{2}\right) = (1, -4)$

3. The answer is C.

4. The answer is A.
Convert to standard form by completing the square.
$x^2 + y^2 - 8x + 2y + 1 = 0$
$x^2 + y^2 - 8x + 2y = -1$
$x^2 - 8x + y^2 + 2y = -1$
$(x^2 - 8x + 16) + (y^2 + 2y + 1) = -1 + 16 + 1$
$(x-4)^2 + (y+1)^2 = 16$
Center: $(4, -1)$, Radius: 4

5. Center = midpoint between $(-2, 0)$ and $(-4, 4)$

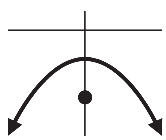
$$= \left(\frac{-2-4}{2}, \frac{0+4}{2}\right) = (-3, 2)$$

Radius = distance between $(-3, 2)$ and $(-2, 0)$

$$= \sqrt{(-2-(-3))^2 + (0-2)^2} = \sqrt{5}$$

$(x+3)^2 + (y-2)^2 = 5$ Standard form
$x^2 + 6x + 9 + y^2 - 4y + 4 = 5$ Multiply out.
$x^2 + y^2 + 6x - 4y + 8 = 0$ Geneal form

6. The answers are A and B.



A) The focus is below the directrix, so the parabola opens down.
B) The vertex, the midpoint between the focus and the directrix, is $(3, 0)$.
C) The axis of symmetry, the line passing through the focus and the vertex, is $x = 3$.
D) $(0, -3)$ is not equidistant from the focus and the directrix, so it is not on the parabola.

7. Let (x, y) be a point on the parabola.
distance to the directrix = distance to the focus
$\sqrt{(x-1)^2 + (y-y)^2} = \sqrt{(x-3)^2 + (y-0)^2}$
$\sqrt{(x-1)^2} = \sqrt{(x-3)^2 + y^2}$ Simplify.
$(x-1)^2 = (x-3)^2 + y^2$ Square both sides.
$x^2 - 2x + 1 = x^2 - 6x + 9 + y^2$ Expand.
$x = \dfrac{1}{4}y^2 + 2$ Sove for x.

8. $(x+2)^2 + (x+2)^2 = 8$ Plug eq2 into eq1.
$2x^2 + 8x = 0$ Write in standard form.
$2x(x+4) = 0$ Sove for x.
$x = 0, x = -4$

$y = 0 + 2 = 2$ Find y when $x = 0$.
$y = -4 + 2 = -2$ Find y when $x = -4$.

Solutions: $(0, 2), (-4, -2)$

9. $2x + (x-1)^2 = 1$ Plug eq1 into eq2.
$x^2 = 0$ Write in standard form.
$x = 0$ Sove for x.

$y = (0-1)^2 = 1$ Find y when $x = 0$.

Solution: $(0, 1)$

10. $x^2 + 3 = 4x + k$ Plug eq1 into eq2.
$x^2 - 4x + 3 - k = 0$ Write in standard form.

The discriminant, $b^2 - 4ac$, must be zero.
$(-4)^2 - 4(1)(3-k) = 0$ Set discriminant = 0.
$k = -1$ Sove for k.

11. $a_1 = 5, d = 4$
$a_n = a_1 + (n-1)d$
$= 5 + 4(n-1)$
$= 4n + 1$
$a_{25} = 4(25) + 1 = 101$

12. $d = 6$
$p = 1 - 6 = -5$
$q = 7 + 6 = 13$
$p + q = -5 + 13 = 8$

13. $a_1 = 5, r = -2$
$a_n = a_1 r^{n-1}$
$= 5(-2)^{n-1}$
$a_6 = 5(-2)^5 = -160$

14. $1, 4, 7, 10, \ldots$
Each term is 3 more than the previous term.
$a_1 = 1, a_n = a_{n-1} + 3$

15. $3, 6, 12, 24, 48, \ldots$
Each term is twice the previous term.
$a_1 = 3, a_n = 2a_{n-1}$

16. The answer is D.
A) $5 + 6 + 7 + \cdots + 17$
B) $5 + 6 + 7 + 8 + 9$
C) $5 + 10 + \cdots + 25$

17. Arithmetic with $a_1 = 2$ and $d = 7$
$a_n = 2 + 7(n-1) = 7n - 5$
$a_{30} = 7(30) - 5 = 205$

$$S_{30} = \frac{30}{2}(a_1 + a_{30}) = \frac{30}{2}(2 + 205) = 3105$$

18. Geometric with $a_1 = 1$ and $r = 5$

$$S_8 = a_1\left(\frac{1-r^8}{1-r}\right) = 1\left(\frac{1-5^8}{1-5}\right) = 97656$$

19. Each year the balance increases by $2500(0.04) =$ \$100. The balance at the end of the first year is \$2,600.
$2600, 2700, 2800, 2900, \ldots$
$a_1 = 2500$ and $d = 100$, so $a_n = 100n + 2500$.
$a_{10} = 100(10) + 2500 = 3500$
The balance will be \$3,500.

20. Each height is 0.5 times the prior one. The height after the first bounce is $40(0.5) = 20$ meters.

$20, 20(0.5), 20(0.5)^2, 20(0.5)^3, \ldots$

$a_1 = 20$ and $r = 0.5$, so $a_n = 20(0.5)^{n-1}$.

$a_5 = 20(0.5)^4 = 1.25$

The ball will rebound to 1.25 meters.

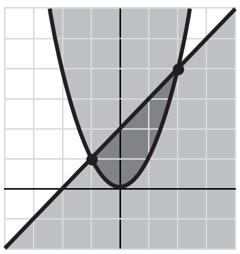
21. $5, 5.5, 6, 6.5, 7, 7.5, 8, \ldots$

$a_1 = 5$ and $d = 0.5$, so $a_n = 0.5n + 4.5$.

$a_{25} = 0.5(25) + 4.5 = 17$

$$S_{25} = \frac{25}{2}(a_1 + a_{25}) = \frac{25}{2}(5 + 17) = 275$$

The total distance is 275 miles.

21. 

The graphs intersect at $(-1, 1)$ and $(2, 4)$.

The maximum possible value of y is 4.