

LESSON 60 Applications of Quadratic Equations

□ REFRESH YOUR SKILLS

(Lesson 7) Solve using the general strategy of solving word problems: 1) define a variable, 2) set up an equation to model the given situation, 3) solve the equation as usual, and then 4) answer what's being asked.

1. The sum of three consecutive even integers is 12. Find the integers.
2. The length of a rectangle is twice its width. The perimeter is 18 feet. Find the dimensions.

(Geometry) The Pythagorean Theorem says that in a right triangle, $c^2 = a^2 + b^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse. Find the missing side.

3. A right triangle has legs of 3 inches and 4 inches. How long is the hypotenuse?
4. A right triangle has a hypotenuse of 13 cm and a leg of 5 cm. How long is the other leg?

□ SOLVING WORD PROBLEMS INVOLVING QUADRATIC EQUATIONS

Word problems involving quadratic equations can also be solved using the general strategy above. Notice that an equation that models the given situation is now quadratic, not linear.

➔ **EXAMPLE** The product of two positive consecutive even integers is 80. Find the integers.

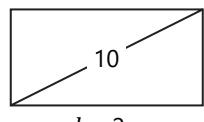
Note that you are to find "positive" integers, so discard any negative solutions.

1. Let x = first even integer
 $x + 2$ = second even integer
2. The product is 80, so $x(x + 2) = 80$.
3. Solve by factoring or by completing the square, and you get $x = 8$ or $x = -10$.
4. The two positive integers are 8 and 10.

➔ **TRY IT 5.** One positive integer is 3 more than another. Their product is 40. Find the integers.

When working with geometry problems, it is helpful to draw a picture. Remember that dimensions must be positive. Discard solutions that result in negative dimensions.

➔ **EXAMPLE** The diagonal of a rectangle is 10 cm. Its length is twice its width. Find the dimensions of the rectangle.

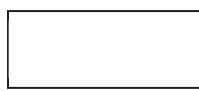


The diagonal is the hypotenuse of the two right triangles.

1. Let x = width of the rectangle
 $2x$ = length of the rectangle
2. Applying the Pythagorean Theorem gives
 $x^2 + (2x)^2 = (10)^2$.
3. Solve by taking the square root of each side, and you get $x = 2\sqrt{5}$ or $x = -2\sqrt{5}$.
4. The width is $2\sqrt{5}$ cm and the length is $4\sqrt{5}$ cm.

➔ **TRY IT 6.** The diagonal of a square measures $5\sqrt{2}$ cm. Find the length of each side.

→ **EXAMPLE** The area of a rectangle is 75 cm^2 . Its length is 5 cm longer than twice its width. Find the dimensions of the rectangle.



$$w = x$$

The area of a rectangle is the length times the width.

1. Let $x = \text{width of the rectangle}$
 $2x + 5 = \text{length of the rectangle}$
2. The area is 75, so $x(2x + 5) = 75$.
3. Solve by factoring or by the quadratic formula, and you get $x = 5$ or $x = -15/2$.
4. The width is 5 cm and the length is 15 cm.

→ **TRY IT 7.** The area of a rectangle is 96 cm^2 . Its length is 4 cm longer than its width. Find the dimensions of the rectangle.

□ **EXERCISE YOUR SKILLS**

For each problem, 1) define a variable, 2) set up an equation, 3) solve the equation, and 4) answer what's being asked.

8. The sum of the squares of two consecutive positive integers is 61. Find the integers.
9. The sum of two positive integers is 10. The sum of their squares is 58. Find the integers.
10. The difference between two positive integers is 3. The sum of the smaller and the square of the larger is 39. Find the integers.
11. A rectangle has a diagonal of 13 cm and a width of 5 cm. Find the length of the rectangle.
12. The hypotenuse of a right triangle is 1 cm longer than one leg and 2 cm longer than the other leg. Find the dimensions of the triangle.
13. The area of a rectangle is 65 square feet. Its length is 2 feet shorter than three times its width. Find the dimensions of the rectangle.
14. A rectangle has a perimeter of 36 inches and an area of 80 square inches. Find the dimensions of the rectangle.
15. An isosceles right triangle, a right triangle with two legs equal in length, has a hypotenuse of $4\sqrt{2}$ feet. Find the perimeter of the triangle.

(CHALLENGE) Solve.

16. A rectangle has a width of 6 cm and a length of 4 cm. A strip of uniform width is added around the rectangle to increase its area by 24 cm^2 . What is the width of the strip?
17. Two cars left an intersection at the same time. Car A traveled north and car B traveled east. When car A traveled 20 miles, the distance between the two cars was 10 miles more than the distance traveled by car B. How far did car B travel?