LESSON 86

1. The answer is D.

$(a+b)^2-(a-b)^2$
$=(a^2+2ab+b^2)-(a^2-2ab+b^2)$
$=a^2+2ab+b^2-a^2+2ab-b^2$
$=4ab$

2. x^4-81

$=(x^2+9)(x^2-9)$	Difference of squares
$=(x^2+9)(x+3)(x-3)$	Difference of squares

3. $4x^3-100x$

$=4x(x^2-25)$	Factor out the GCF.
$=4x(x+5)(x-5)$	Difference of squares

4. x^4-x^3+x-1

$=x^3(x-1)+(x-1)$	Factor by grouping.
$=(x-1)(x^3+1)$	
$=(x-1)(x+1)(x^2-x+1)$	Sum of cubes

5. The solutions are $x=0$ with multiplicity 1, $x=-1$ with multiplicity 3, and $x=1$ with multiplicity 2.

6. $x^3=x-2x^2$

$x^3+2x^2-x=0$	Standard form
$x(x^2+2x-1)=0$	Factored form
$x=0$ or $x^2+2x-1=0$	Zero-product property
$x=0, x=\frac{-2\pm\sqrt{8}}{2}$	Solve for x.
$x=0, x=-1\pm\sqrt{2}$	Simplify.

7. $x^3-2x^2+x-2=0$

$x^2(x-2)+(x-2)=0$	Factor by grouping.
$(x-2)(x^2+1)=0$	Factored form
$x-2=0$ or $x^2+1=0$	Zero-product property
$x=2, x^2=-1$	Solve for x.
$x=2, x=\pm i$	

Sum $=2+i-i=2$

8. $(x-1)\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)=0$ Factored form
$(x-1)(x^2-2)=0$ Multiply out.
$x^3-x^2-2x+2=0$ Standard form

9. Sum of $1\pm 2i=2$, Product of $1\pm 2i=5$
Quadratic factor with roots $1\pm 2i=x^2-2x+5$

$x(x^2-2x+5)=0$	Factored form
$x^3-2x^2+5x=0$	Standard form

10. $x(x+1)^2(x-1)=0$ Factored form
$x(x^2+2x+1)(x-1)=0$ Multiply out.
$(x^3+2x^2+x)(x-1)=0$
$x^4+x^3-x^2-x=0$ Standard form

11. The answer is C.

12. The answer is D.
The zeros are 0, 2, and 4, so eliminate A and C.
The leading coefficient is negative because the right end of the graph goes down, so choose D.

13. The answer is D.
The zeros are -3 and 1, so eliminate A and C.
Both zeros have an even multiplicity, so choose D.

14. The answers are A and D.

B) The graph touches the x-axis at $x=-4$ because $x=-4$ has an even multiplicity of 2.

C) Test a point. $f(1)=-25<0$

D) The degree is 6 (even). The leading coefficient is 1 (positive). Both ends go up.

15. $x^5-4x^4+4x^3=0$ Set $f(x)=0$.
$x^3(x^2-4x+4)=0$ Factor out the GCF.
$x^3(x-2)^2=0$ Perfect square trinomial

$x=0$ (multiplicity 3, crosses the x-axis)
$x=2$ (multiplicity 2, touches the x-axis)

16. The answer is C.

A) $f(x)=x(x-2)(x+3)$
B) $f(x)=x(x+2)(x-3)$
C) $f(x)=-x(x+2)(x-3)$
D) $f(x)=-x(x-2)(x+3)$

The zeros are 0, -2, and 3, so eliminate A and D.
The leading coefficient is negative, so choose C.

17. $f(x)=ax(x-2)(x-3)$ Factored form
$6=a(1)(1-2)(1-3)$ Plug in $(1,6)$.
$6=2a$ Solve for a.
$a=3$

$f(x)=3x(x-2)(x-3)$ Plug in a.
$f(x)=3x^3-15x^2+18x$ Standard form

18. $f(x)=ax^2(x-4)^2$ Factored form
$-8=a(2^2)(2-4)^2$ Plug in $(2,-8)$.
$-8=16a$ Solve for a.
$a=-1/2$

$f(x)=-\frac{1}{2}x^2(x-4)^2$ Plug in a.

$f(x)=-\frac{1}{2}x^4+4x^3-8x^2$ Standard form

19. By the Factor Theorem $p(1)=0$
$p(1)=1^3-1^2-1+c=0$
Solve for c, and you get $c=1$.

$k=p(-1)=(-1)^3-(-1)^2-(-1)+c$
$=-1-1+1+1=0$