LESSON 98 Applications of Exponentials

□ **REFRESH YOUR SKILLS** ···

(Lesson 97) The function $y = 10000(0.96)^t$ models the population of a town after t years.

1. Is this exponential growth or decay?

2. By what percent does the population change each year?

□ **SOLVING WORD PROBLEMS INVOLVING EXPONENTIAL GROWTH AND DECAY** ·····························

When solving a word problem involving exponential growth or decay, first identify the initial value, a, and the growth or decay factor, b, to write an exponential function of the form $y = ab^x$ that models the situation. Then you can use the function to answer whatever is being asked.

Compound interest is a good example of exponential growth. **Compound interest** is calculated on the **principal**, the initial amount of money, plus any accumulated interest.

➔ **EXAMPLE** Adam put $800 in an account that earns 4% interest compounded annually. a) Write an exponential function to model the balance of the account, y, after t years. b) What will be the balance after 5 years?

> Notice that this is exponential growth because $b > 1$. The balance will grow exponentially.

a. a = initial balance = 800
  b = 100% + 4% = 104% = 1.04 because
  new balance = previous + previous × 0.04
           = previous × (1 + 0.04)
           = previous × 1.04
  The function is $y = 800(1.04)^t$

b. When $t = 5$, $y = 973.32...$
  The balance will be about $973.

➔ **TRY IT 3.** The starting salary for Alex's job is $50,000 a year. He is to get a 5% raise each year.

  a. Write an exponential function to model Alex's salary, y, after t years.

  b. What will his salary be after 7 years? Round to the nearest dollar.

➔ **EXAMPLE** The value of a car is $20,000. It loses 20% of its value every year. a) Write an exponential function to model the value of the car, y, after t years. b) What will the value be after 6 years?

> Notice that this is exponential decay because $0 < b < 1$. The value will decrease exponentially.

a. a = initial value = 20000
  b = 100% − 20% = 80% = 0.8 because
  new value = previous − previous × 0.2
         = previous × (1 − 0.2)
         = previous × 0.8
  The function is $y = 20000(0.8)^t$.

b. When $t = 6$, $y = 5242.88...$
  The value will be about $5,243.

➔ **TRY IT 4.** The population of a city is **50,000** and is decreasing at a rate of 5% per year.

  a. Write an exponential function to model the population, y, after t years.

  b. What will be the population after 7 years? Round to the nearest whole number.

Be careful finding exponents. Remember, in the form $y = ab^x$, the y-value is multiplied by b each time x increases by 1.

➜ **EXAMPLE** A culture of 100 bacteria doubles, or increases by a factor of 2, every three hours. a) Write an exponential function to model the number of bacteria, y, after t hours. b) How many bacteria will be there after 12 hours?

> Notice that the y-value doubles at t = 3, 6, 9, ….
> In other words, the y-value dobules at $t/3$ = 1, 2, 3, …..

a. a = initial value = 100
   b = growth factor = 2
   $t/3$ = exponent because
      the y-value is multiplied by 2
      each time $t/3$ increases by 1.
   The function is $y = 100(2)^{t/3}$.

b. When t = 12, y = 1600.
   There will be 1,600 bacteria.

➜ **TRY IT 5.** The bear population in an area is 300 and is decreasing at a rate of 40% every 10 years.

    a. Write an exponential function to model the bear population, y, after t years.

    b. What will be the bear population after 30 years? Round to the nearest whole number.

☐ **EXERCISE YOUR SKILLS** ···

Write an exponential function that models the situation, identify your function as exponential growth or decay, then answer what's being asked. Round to the nearest whole number.

6. Melissa put $3,000 in her account that earns 4% interest compounded annually. What will be the balance of her account after 8 years?

7. Jessica earns $60,000 a year working as a programmer. Each year she receives a 6% raise. What will her salary be after 10 years?

8. As a car gets older, its resale value goes down by 10% each year. Alex bought a new car for $25,000. What will be the value of the car after 6 years?

9. The population of a town is 30,000 this year, and it is expected to decline at a rate of 4% per year. What will be the population after 8 years?

10. The atmospheric pressure at sea level is 1,013 millibars. The pressure decreases by 14% for every kilometer higher up in altitude. What is the pressure at an altitude of 5 km?

11. During a science experiment, Jessica found that bacteria double every two hours. There were 50 bacteria in the beginning. How many bacteria will be there after 10 hours?

12. A new website had a total of 300 hits on the first day. The traffic is estimated to increase by 20% every five days. How many hits will the website have after 30 days?

13. (CHALLENGE) Sandar invested $10,000 in a mutual fund that earns 2% compound interest every six months. How much will be in the funds in 20 years?