LESSON 180

1. 8	**2.** 2	**3.** -4	**4.** -1
5. A, C	**6.** 16	**7.** 5	**8.** A, B, E
9. B	**10.** 1	**11.** 1, 4	**12.** B
13. 4 hrs	**14.** 15	**15.** -3	**16.** 9
17. 4	**18.** A	**19.** $x+2$	**20.** 7
21. B	**22.** 12 hrs	**23.** $2\sqrt{2}$	**24.** 5
25. -1	**26.** 1.458	**27.** 1,335	**28.** C
29. 900	**30.** $1/5$		

Worked-out solutions:

1. $|x-3|+1=6$

$\lvert x-3\rvert=5$	Isolate the absolute value.
$x-3=5$ or $x-3=-5$	Rewrite as two equations.
$x=8, x=-2$	Solve each equation.

The positive solution is $x=8$.

2. First, find the equation of the line.

$m=\frac{3-6}{1-0}=-3$	Find the slope using the slope formula.
$b=6$	$(0,6)$ is the y-intercept.
$y=mx+b$	Slope-intercept form
$y=-3x+6$	Plug in m and b.

Then, find the x-intercept.

$0=-3x+6$	Set $y=0$.
$x=2$	Solve for x

3.

$6x+2y=2$	eq1 × 2
$x+2y=7$	eq2
$5x=-5$	Subtract the equations.
$x=-1$	Solve for x.
$3(-1)+y=1$	Plug x into eq1.
$y=4$	Solve for y.

$ab=(-1)(4)=-4$

4. $(x+1)^2=x+3$

$x^2+2x+1=x+3$	Expand each side.
$x^2+x-2=0$	Write in standard form.

$m+n=$ sum of roots $=-b/a=-1/1=-1$

5. A) The graph crosses the x-axis at $x=2$ because $x=2$ has an odd multiplicity of 1.

B) The graph touches the x-axis at $x=-1$ because $x=-1$ has an even multiplicity of 2.

C) Test a point. $f(0)=-4<0$

D) The leading term is $2x^3$. The degree is 3 (odd). The leading coefficient is 2 (positive). The left end of the graph goes down and the right end goes up.

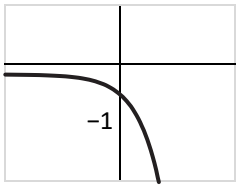
6. $(8x^2)(2x^{-3})^{-2}=(2^3x^2)(2^{-2}x^6)=2^{3-2}x^{2+6}=2x^8$

$ak=2(8)=16$

7. $3^{x-1}=9^{x-3}$

$3^{x-1}=(3^2)^{x-3}$	Rewrite using base 3.
$3^{x-1}=2^{2(x-3)}$	Exponent rules
$x-1=2(x-3)$	One-to-one property
$x=5$	Solve for x.

8.



Sketch the graph.
1. Graph $y=e^x$
2. Reflect over the x-axis to get $f(x)=-e^x$.

A) The y-intercept is $f(0)=-e^0=-1$.

B) The asymptote of $y=e^x$ is $y=0$. The reflection does not change the asymptote

C) As x increases, $f(x)$ decreases.
As x decreases, $f(x)$ approaches 0.

D) The graph is in Quadrants III and IV.

9. Compound interest is exponential growth. $B(t)$ must be an exponential function, so eliminate C and D.

Each year's balance is 104% of the previous year's balance, or 1.04 times the previous year's balance. The growth factor is 1.04, so choose B.

10.

$\log_3 1=0$	Zero rule
$\log_3 3=1$	Identity rule
$\log_3 27=\log_3 3^3=3$	Use $\log_b b^n=n$.

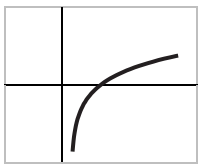
$\log_3 1-2\log_3 3+\log_3 27=0-2(1)+3=1$

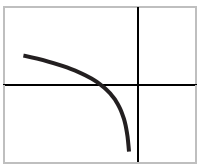
11. $\ln x+\ln 9=2\ln(x+2)$

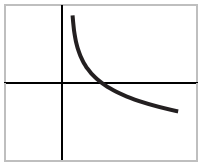
$\ln 9x=\ln(x+2)^2$	Logarithm rules
$9x=(x+2)^2$	One-to-one property
$x^2-5x+4=0$	Standard form
$(x-1)(x-4)=0$	Solve for x.
$x=1, x=4$	

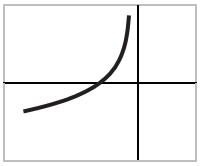
Both solutions are valid.

12. $y=\log_2 x$ is reflected over the y-axis.

A) 

B) 

C) 

D) 

13. The initial amount is 100 mg.

The decay factor is $100\% - 17\% = 83\% = 0.83$.

The function $y = 100(0.83)^t$ models the amount of caffeine in the body t hours after drinking a cup of coffee.

Use the function to find t when $y = 50$.

$50 = 100(0.83)^t$

$(0.83)^t = 1/2$

$t = \log_{0.83}(1/2) = \dfrac{\ln(1/2)}{\ln 0.83} = 3.72000\ldots$

It will take about 4 hours.

14. $\sqrt[5]{32} + 64^{2/3} - 81^{1/4}$

$= \sqrt[5]{2^5} + (4^3)^{2/3} + (3^4)^{1/4}$

$= 2 + 4^2 - 3 = 15$

15. $x^{2/5}(x^3)^{2/5} = x^{2/5}x^{6/5} = x^{8/5} = \sqrt[5]{x^8}$

$m - n = 5 - 8 = -3$

16. $x - 4 = \sqrt{2x + 7}$

$(x-4)^2 = 2x + 7$	Square both sides.
$x^2 - 10x + 9 = 0$	Write in standard form.
$(x-1)(x-9) = 0$	Solve for x.

$x = 1, x = 9$

$x = 1$ is extraneous, so the solution is $x = 9$.

17. $(4x+3)^{3/2} = 2^3$

$\left[(4x+3)^{3/2}\right]^{2/3} = (2^3)^{2/3}$	Raise to the reciprocal power, and solve for x.
$4x + 3 = 2^2$	
$4x = 1$	
$x = 1/4$	
$x^{-1} = (1/4)^{-1} = 4$	Negative exponent rule

18. $(-3, -1)$ is on the graph, so eliminate B and D.

The graph does not involve a reflection, so choose A.

19. Multiply the numerator and denominator by the LCD of all fractions in the numerator and denominator.

$$\frac{1 - \frac{4}{x^2}}{\frac{1}{x} - \frac{2}{x^2}} = \frac{x^2\left(1 - \frac{4}{x^2}\right)}{x^2\left(\frac{1}{x} - \frac{2}{x^2}\right)}$$

$$= \frac{x^2 - 4}{x - 2} = \frac{(x+2)(x-2)}{x-2} = x + 2$$

20. $\dfrac{2}{x} - \dfrac{1}{4} = \dfrac{2}{x^2 + x}$

$\dfrac{2}{x} - \dfrac{1}{4} = \dfrac{2}{x(x+1)}$	Restrictions: $x \neq 0, -1$ LCD $= 4x(x+1)$
$8(x+1) - x(x+1) = 8$	Multiply both sides by the LCD, then solve for x.
$x^2 - 7x = 0$	
$x(x-7) = 0$	
$x = 0, x = 7$	

$x = 0$ is extraneous, so the solution is $x = 7$.

21. $f(x)$ is $y = 1/x$ shifted left 3 units and up 2 units.

The vertical asymptote of $y = 1/x$ at $x = 0$ is shifted left 3 unit to $x = -3$.

The horizontal asymptote of $y = 1/x$ at $y = 0$ is shifted up 2 units to $y = 2$.

22. $x =$ pipe B's time alone

Pipe A's rate + Pipe B's rate = combined rate, so

$$\frac{1}{4} + \frac{1}{x} = \frac{1}{3}$$

Solve for x, and you get $x = 12$.

It will take 12 hours.

23. Convert to standard form by completing the square.

$x^2 + y^2 + 4x - 6y + 5 = 0$

$x^2 + y^2 + 4x - 6y = -5$

$x^2 + 4x + y^2 - 6y = -5$

$(x^2 + 4x + 4) + (y^2 - 6y + 9) = -5 + 4 + 9$

$(x+2)^2 + (y-3)^2 = 8$

Radius $= \sqrt{8} = 2\sqrt{2}$

24.

$(x-2)^2 + (x-3)^2 = 25$	Plug eq2 into eq1.
$2x^2 - 10x + 12 = 0$	Write in standard form.

Sum of roots $= -b/a = -(-10)/2 = 5$

25. The graphs intersect at one point.

= The system has one real solution.

$2x + k = x^2 - 2x + 3$	Plug eq2 into eq1.
$x^2 - 4x + 3 - k = 0$	Write in standard form.

The discriminant, $b^2 - 4ac$, must be zero.

$(-4)^2 - 4(1)(3-k) = 0$	Set discriminant $= 0$.
$k = -1$	Solve for k.

26. This is a geometric sequence with $a_1 = 2$ and $r = 3$.

$a_n = a_1 r^{n-1} = 2(3)^{n-1}$

$a_7 = 2(3)^6 = 1458$

27. This is an arithmetic sequence with $a_1 = 1$ and $d = 3$.

$a_n = 1 + 3(n-1) = 3n - 2$

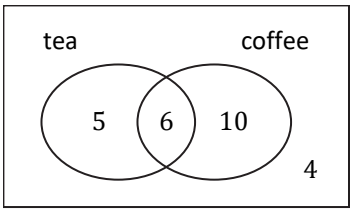
$a_{30} = 3(30) - 2 = 88$

$$S_{30} = \frac{30}{2}(a_1 + a_{30}) = \frac{30}{2}(1 + 88) = 1335$$

28. Mean $=$ sum divided by count $= 4.5$
Median$=$ middle value when ordered $= 4.5$

29. Estimate $=$ population size $\cdot$ sample proportion
$1200(60/80) = 900$
A reasonable estimate is 900 employees.

30.



P(tea only)
$= 5/25$
$= 1/5$